'02 Honors Trigonometry

EXAM CHAPTER 3

No Homework *again!*
... and to think we get honors credit ...
11/1

Read ONLY: Pgs 109-114 and pgs 122 - 125

• • • • • • • • • • • • • • • •

Do section I on back of HW sked # 1
1st half of 1st lecture
11/4

2 Section II on back of sked #2

Finish 1st lecture
11/5

3 Prove:

$$\cos\left(\sin^{-1} u\right) = \sqrt{1-u^2}$$

4 Pg 126 1 – 7, 19 – 24

Lecture # 2
Evaluate inverse functions fm text 11/6

5 Pg 121 7 – 21
Section V on back of sked
Lecture # 3
then have hw #4 put on boards by students
11/7

6 Section III back of sked
7 R/C pg 137
Lecture # 4 proof of:

$$\cos^{-1} x = \sec^{-1}\left(\frac{1}{x}\right)$$

$$\sin\left(\tan^{-1} u\right) = \frac{u}{\sqrt{1+u^2}}$$
11/8

School Holiday

Start work on # 8 Section IV on back of sked.
11/11

Finish/ReDo Section IV on back of sked
Quest for Quiz Points???
(there are 5 other...)
11/12

"FREEDOM RINGS"

It almost makes you want to march!
Value: **10 assignments!!!**
Lecture Below 11/13

Have at least 1 - 20 done by the end of the day.

Lecture # 5
in notebook
11/14

Continue having fun in Freedomland.
Une autre la quizzette??
11/15

When was the last time you looked at the "exam" door?

You should have completed 1 - 60 by the beginning of the period tomorrow!
11/18

"...there are 5 other functions...."

$a(x) = \cos^{-1} x$

$D_a \quad -1 \le x \le 1$

$R_a \quad 0 \le \cos^{-1} x \le \pi$
11/19

including "memory device" and graph of the function

On day beginning of Freedom Rings lecture showing solution of:

$\sin 2x = \sin x$

$\sin x - \sqrt{3}\cos x = 1$

extra root eliminated
11/20

Review

Be prepared to put up last year's exam. Come to class prepared!
11/21

Exam Ch 4
11/22

$$\tan^{-1} x \neq \frac{\sin^{-1} x}{\cos^{-1} x}$$

I Find the value of each of the following:

a) $\cos^{-1}\frac{\sqrt{3}}{2}$ b) $\sin^{-1}\frac{1}{\sqrt{2}}$ c) $\sin^{-1}\frac{1}{2}$ d) $\cos^{-1}\left(-\frac{1}{2}\right)$ e) $\cos\left(\cos^{-1}\frac{1}{3}\right)$ f) $\sin\left(\sin^{-1}\frac{1}{5}\right)$

g) $\cos\left(\sin^{-1}\left(-\frac{4}{5}\right)\right)$ h) $\sin\left(\cos^{-1}\left(-\frac{5}{13}\right)\right)$ i) $\arcsin(\sin 3)$ j) $\arccos(\cos 5)$

II Determine ALL values of u such that: a) $\left(\sqrt{u^2}\right) = u$ b) $\left(\sqrt{u}\right)^2 = u$

c) $\cos\left(\cos^{-1} u\right) = u$ d) $\cos^{-1}(\cos u) = u$ e) $\sin\left(\sin^{-1} u\right) = u$ f) $\sin^{-1}(\sin u) = u$

III a) Pg 121 problem 22 b) When is this true? $\csc x = \dfrac{1}{\sin x}$ c) Determine $\sin^{-1} x = \csc^{-1}$?

d) complete: $\cos\left(\tan^{-1} u\right) = \cdots$ e) Simplify: $\sin\left(\sin^{-1}\frac{5}{13} + \tan^{-1}\frac{1}{2}\right)$ $\dfrac{22\sqrt{5}}{65}$

f) Simplify: $\cos\left(\tan^{-1}\frac{12}{13} + \sin^{-1}\frac{1}{2}\right)$ $\dfrac{13\sqrt{939} - 12\sqrt{313}}{626}$

IV a) Do page 126 problems 25, 26, and 27

b) verify : (you may work with *both* sides.) : (1) $\cot^{-1} x = \tan^{-1}\frac{1}{x}\ ;\ x > 0$ (2) $\tan^{-1} x = \frac{\pi}{2} - \cot^{-1} x$

c) True or false? : $\cot^{-1} x = \tan^{-1}\frac{1}{x} + \pi\ ;\ x < 0$ (Hint: graph $a(x) = \cot^{-1} x$ and $b(x) = \tan^{-1} x$ on the same axis.) Explain.

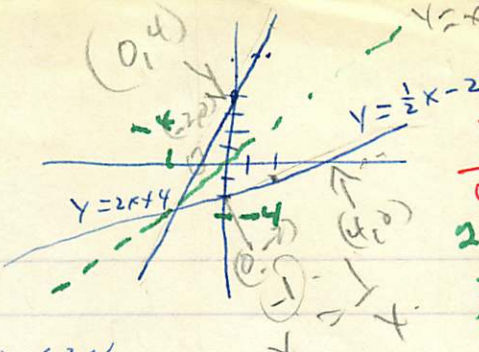
V Determine ALL values of u such that:

a) $\tan\left(\tan^{-1} u\right) = u$ b) $\tan^{-1}(\tan u) = u$ c) $\csc^{-1}(\csc u) = u$ d) $\csc\left(\csc^{-1} u\right) = u$

e) $\sec\left(\sec^{-1} u\right) = u$ f) $\sec^{-1}(\sec u) = u$ g) $\cot\left(\cot^{-1} u\right) = u$ h) $\cot^{-1}(\cot u) = u$

This lecture takes 2 days!

1ST lecture Chapter 4



to get intersection set $2x+4 = \frac{1}{2}x-2$

$\frac{3}{2}x = -6$

$x = -4$

I Given $f(x) = 2x+4$

① Graph consider $y = 2x+4$

② Determine inverse of $f(x) = 2x+4$

2 STEP Process

$x = 2z+4$

$2z = x-4$

$z = \frac{1}{2}x-2$ = Inverse.

$f^{-1}(x) = \frac{1}{2}x-2$

1:15 start

II what we have is a inverse of a fctn.

A. Notice it is merely a reflection across $y=x$.

Because we swapped the x and y.

B. Notice ANY value in Domain of original fctn became a value in the Range of the inverse fctn. This is clear in the method of changing a given fctn to its inverse. (ie substituting x for y and y for x) See pages 25, 26, 27

III Consider the fctn $y = x^2$.

A. Graph

B. Determine inverse

C. Graph inverse. (use reflections over $y=x$)

D. Notice inverse is not a fctn. doesn't pass vertical line test.

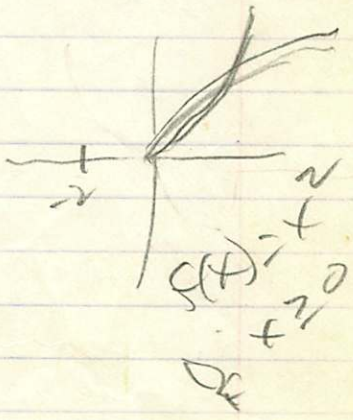
IV Considis the fctn $y = \cos x$

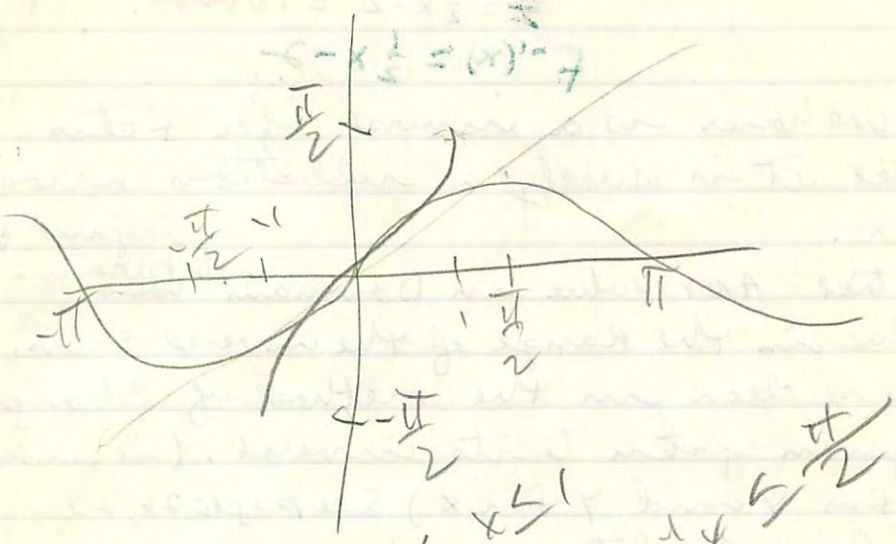
A. Graph

B. Determine inverse

C. Graph inverse

D. Do we have a fctn? NO

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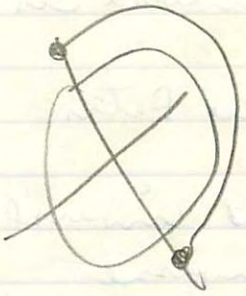
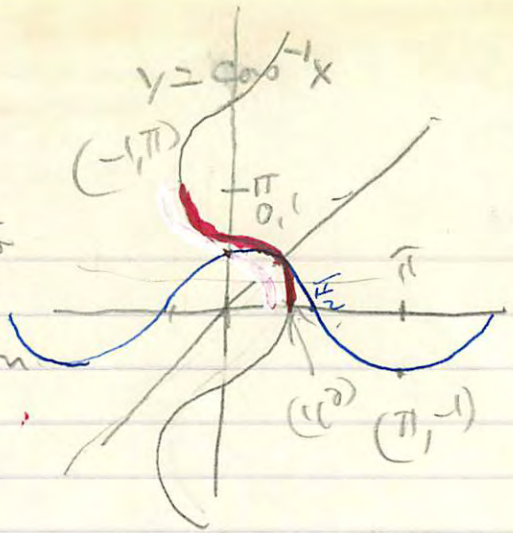




II we would like to work with a function rather than a relation if possible.

A. Notice all values in the Range of $y = \cos^{-1} x$ are positive.

B. In order to be concerned only about the smallest possible numbers the choice of Period to consider is clearly $0 \leq y \leq \pi$. Had we chosen $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$ to achieve smallest values (thinking in terms of absolute value), we would still not have a function.

III Given $y = \cos x$

A. we think of x as arc length on a circle

B. Therefore when we say

$$\cos^{-1} x = y$$

we can say that y is an arc length.

Thus the recognition of arc $\cos x \equiv \cos^{-1} x$

C. Conclusions:

NOTICE $\cos^{-1} x \neq \frac{1}{\cos x}$

$(\cos x)^{-1} = \frac{1}{\cos x}$

1. Definition: $\cos^{-1} x = \{(x, y) \; y = \cos^{-1} x\}$

2. Domain $-1 \leq x \leq 1$

3. Range $0 \leq y \leq \pi$

VII $\sin^{-1} x = y$

Using the ideas above we arrive at;

A Definition $\sin^{-1} x = \{(x, y) \; y = \sin^{-1} x\}$

B Domain $-1 \leq x \leq 1$

C Range $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

1st Lecture

VII Examples

Note: Students should have these copied as notes. There will be reference to them later.

Do not Allow open Books

Solve for a

1) $\cos^{-1}\frac{\sqrt{3}}{2} = a$

$$\boxed{\cos\left(\cos^{-1}\frac{\sqrt{3}}{2}\right) = \cos(a)}$$

(Since Domain Cos is $\mathbb{R}$ and $\cos^{-1}\frac{\sqrt{3}}{2} \in \mathbb{R}$ all is well defined)

$\frac{\sqrt{3}}{2} = \cos a$

Range of $\cos^{-1}$

Instead of this... think "Cosine of what is $\frac{\sqrt{3}}{2}$?"

1st time through show only through VII 2. Assign same HW. 2nd Lecture begin VII 4 to end. Same HW. Do not collect HW. Have students also do #2.

Now what values of a satisfy the abv equation?

$a = \frac{\pi}{6}$ or $a = \frac{11\pi}{6}$

Consider what a is originally supposed to stand for.

Range of $\cos^{-1}$

Now, we can say $a = ?\ \frac{\pi}{6}$ since $\frac{11\pi}{6}$ is not in Range of $\cos^{-1}$.

2) $\cos^{-1}\left(-\frac{1}{2}\right) = a$

$\cos\left(\cos^{-1}\left(-\frac{1}{2}\right)\right) = \cos a$ (Same arguement as above).

$-\frac{1}{2} = \cos a$

a could be $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$.

again we choose $a = \frac{2\pi}{3}$ since $\frac{4\pi}{3}$ is not in ~~domain~~ Range of $\cos^{-1}(u)$

END of "1st Day's Lecture".

method #2.

we don't know $\cos(\sin^{-1}\frac{1}{2})$

but $s^2+c^2=1$ so $\cos^2 t = 1-\sin^2 t$

$\cos t = \pm\sqrt{1-\sin^2 t}$

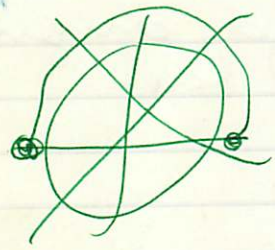
$$\cos(\sin^{-1}\tfrac{1}{2}) = \pm\sqrt{1-\sin^2(\sin^{-1}\tfrac{1}{2})}$$

$$\pm\sqrt{1-\tfrac{1}{4}}$$

$$\pm\sqrt{\tfrac{3}{4}}$$

$$\cos(\sin^{-1}\tfrac{1}{2}) = \pm\frac{\sqrt{3}}{2}$$

but



and cosine of something in quad I or IV is positive so

$$\cos(\sin^{-1}\tfrac{1}{2}) = \frac{\sqrt{3}}{2}.$$

OR

method #1 $\sin^{-1}\frac{1}{2} = a$ so $a = \frac{\pi}{6}$ so $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

use this version first ~~for~~ ~~then~~ show this version

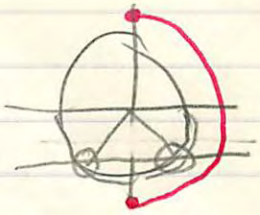
~~this but this version~~

then example 5 is possible using 2nd method

3) $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = a$ (Same argument)

$$\sin\left(\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right) = \sin a$$

$$-\frac{\sqrt{3}}{2} = \sin a$$



$a = \frac{4\pi}{3}$ $a = \frac{5\pi}{3}$.

NEITHER are allowed in Range of $\sin^{-1}$.
Notice however $\sin\frac{5\pi}{3} = \sin\frac{-\pi}{3}$

and $-\frac{\pi}{3}$ is in range $\therefore$ $-\frac{\pi}{3} = a$.

4) $\sin^{-1} 1 = a$

$$\sin(\sin^{-1} 1) = \sin a$$
$$1 = \sin a$$

Clearly $a = \frac{\pi}{2}$ (Notice it is in the Range of $\sin^{-1}$.

4) $\cos(\sin^{-1}\frac{1}{2}) = a$ $-\frac{\pi}{2} \le b \le \frac{\pi}{2}$

Let $\sin^{-1}\frac{1}{2} = b$ then $\frac{1}{2} = \sin b \Rightarrow b = \frac{\pi}{6}$
so $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

5) $\sin(\arccos\frac{3}{5}) = a$ $0 \le b \le \pi$
Let $\arccos\frac{3}{5} = b$ then $\cos b = \frac{3}{5}$

$\sin^2 b + \cos^2 b = 1 \Rightarrow \sin^2 b + \frac{9}{25} = 1 \Rightarrow$

$\sin^2 b = \frac{16}{25} \Rightarrow \sin b = \pm\frac{4}{5}$

Go back to restrictions I imposed.

We know b is in either 1st or 2nd Quadrant (Def of Range of $\cos^{-1}$) and that $\cos b = \frac{3}{5}$ is in either 1st or 4th Quad. Since our discussion is only valid in the areas

for 2nd days: HW —

consider:

for what values of u does this hold:

$$\left(\sqrt{u}\right)^2 = u$$

ANS: only true if $u \geq 0$

this is similar to the 4 equations

where both 1st Lecture cases are true we are in the 1st Quadrant only.

Since this is true, $\sin b = \pm\frac{4}{5}$ can be reduced to simply $\sin b = \frac{4}{5}$.

Questions: what were we originally trying to find?

$\sin(\cos^{-1}\frac{3}{5})$? but we let $b = \cos^{-1}\frac{3}{5}$

So what we were trying to find was $\sin b$.

we have found it $\sin b = \frac{4}{5}$.

VIII Conclusion;

A, we have made use of:

$\cos(\cos^{-1} u) = u$ and $\sin(\sin^{-1} u) = u$ in several of the above examples.

③ Explain fully seeming contradictions.

HW

1. Sketch fctns $\cos^{-1}$, $\sin^{-1}$ mark in values to illustrate R+D.
2. Pblms pg 143 #5-8, 13-18
3. Consider for what values of u is the following true.
   a) $\cos^{-1}(\cos u) = u$
   b) $\cos(\cos^{-1} u) = u$
   c) $\sin^{-1}(\sin u) = u$
   d) $\sin(\sin^{-1} u) = u$.

HW Pg 143

(5) $\cos^{-1} \frac{\sqrt{3}}{2}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}$

(6) $\sin^{-1} \frac{\sqrt{2}}{2}$

$\sin x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}$

(7) $\sin^{-1}\left(-\frac{1}{2}\right)$

$\sin x = -\frac{1}{2}$

$x = -\frac{\pi}{6}$

(8) $\cos^{-1}\left(-\frac{1}{2}\right)$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}$

(9) $\sin^{-1} .994$ $\quad \sin x = .994$

$.53 \cdot \frac{\pi}{2} = .465\pi \approx 1.46 \quad \pi = 3.14$

(10) $\cos^{-1} .233$

$\cos x = .85 \cdot \frac{\pi}{2} = .425\pi = 1.34$

(11) $\cos^{-1} .523$

$\cos x = .65 \cdot \frac{\pi}{2} = .325\pi \approx 1.02$

(12) $\sin^{-1} .876$

$\sin x = .65 \cdot \frac{\pi}{2} = .34\pi = 1.07$

(13) $\cos(\cos^{-1}\frac{1}{3})$ since cos is inverse of $\cos^{-1}$

$\cos(\cos^{-1})\frac{1}{3} = \frac{1}{3}$

(14) $\sin(\sin^{-1}\frac{1}{5}) = \frac{1}{5}$

(15) $\cos[\sin^{-1}(-\frac{4}{5})] =$ $\quad \cos^2 y + \frac{16}{25} = 1 \quad \cos^2 y = \frac{9}{25}$

$\sin^{-1}(-\frac{4}{5}) = y$ $\quad \cos y = \frac{3}{5}$

$\sin y = -\frac{4}{5}$ $\quad \cos(\sin^{-1}(-\frac{1}{5})) = \frac{3}{5}$

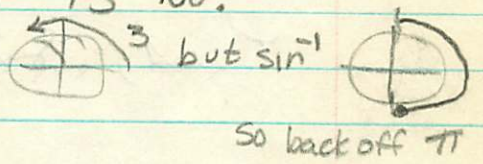
(16) $\sin(\cos^{-1}(-\frac{5}{13})] =$ $\quad \sin^2 y + \frac{25}{169} = 1$

$\cos^{-1}(-\frac{5}{13}) = y$ $\quad \sin^2 y = \frac{144}{169}$

$\cos y = -\frac{5}{13}$ $\quad \sin y = \frac{12}{13}$

(17) arc $\sin(\sin 3) = 3$ } Same as #13 No.

(18) arc $\cos(\cos 5) = 5$ } NO!



but $\sin^{-1}$

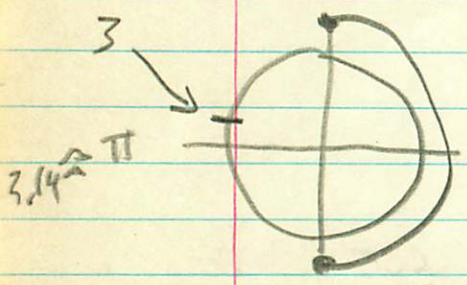
So back off π

(19) $\sin^{-1}(\cos\frac{\pi}{2}) = \sin^{-1}(0) \Rightarrow \sin x = 0$

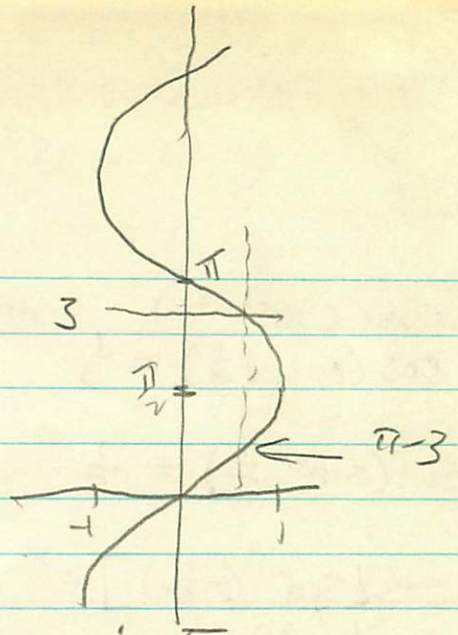
Here we use the fact of the range of $\sin^{-1}$ $\quad x = 0$

This is $\sin^{-1}$ only place it is 0 is at $\theta = 0$ not at $\theta = \pi$

I $\sin^{-1}(\sin 3) = a$



↑ this is a vertical distance measured from x-axis to the spot with arc length 3,



$\sin 3$ is vertical height from x axis to spot on unit circle which has radian measure 3.

this SAME height is found by going π from (1,0) and returning 3,

ie $\pi - 3$,

Thus, $\sin 3 \overset{\text{should}}{=} \sin(\pi - 3)$

$= \sin\pi\cos 3 - \sin 3\cos\pi$

$= 0 - (-\sin 3)$

$\sin 3 = \sin 3$

So we can substitute $\sin(\pi - 3)$ for $\sin 3$.

$\sin^{-1}(\sin(\pi - 3)) = a$

NOW $\sin^{-1}$ undoes $\sin$ because $\pi - 3$ is within the domain of $\sin^{-1}$ to yield $\pi - 3 = a$

1st day's HW

I Begin by reading answers to HW.

A) $\frac{\pi}{6}$ E) $\frac{1}{3}$

B) $\frac{\pi}{4}$ F) $\frac{1}{5}$ G) $\frac{3}{5}$

C) $\frac{\pi}{6}$ H) $\frac{12}{13}$

D) $\frac{2\pi}{3}$ I) $\pi - 3$

J) $2\pi - 5$

use ← to give discussion in proper direction

There should be some flack about Problems 17, 18. This should make students forget the last sections of HW for awhile. Allow about 5 minutes of arguments

II. Last night's HW

Go back into HW. $(\sqrt{u})^2 = u$ then $(\sqrt{u})^2 = u$

II

a) $\cos(\cos^{-1} u) = u \quad |u| \le 1$

b) $\sin(\sin^{-1} u) = u \quad |u| \le 1$

c) $\cos^{-1}(\cos u) = u$

1. as far as the domain of cos is concerned $u \in \mathbb{R}$.
2. as for $\cos^{-1}$ however, the range of $\cos u$ must be within Domain of $\cos^{-1}$. It is since range of $\cos u$ is $|\cos u| \le 1$.
3. u must <u>also</u> be in Range of $\cos^{-1}$! If u is not in Domain of $\cos^{-1}$ it will be in one of the periods of the relation of $\cos^{-1}$. ∴ we must back it down to the f'ctn range of the f'ctn $\cos^{-1}$.

Thus $0 \le u \le \pi$.

WTP $\sin(\cos^{-1}u) = \sqrt{1-u^2}$

Let $\cos^{-1}u = b \quad |u| \le 1 \quad 0 \le b \le \pi$

$\cos(\cos^{-1}u) = \cos b$

$u = \cos b$

because u is in Parametrized

$u^2 = \cos^2 b$

$u^2 = 1 - \sin^2 b$

$\sin^2 b = 1 - u^2$

$\sin b = \pm\sqrt{1-u^2}$

$\sin(\cos^{-1}u) = \pm\sqrt{1-u^2}$

Since b Element in 1ST and 2nd Quadrants,

$\sin b > 0$ ∴ we drop the $\pm$ and keep that

$\sin(\cos^{-1}a) = \sqrt{1-u^2}$

QED,

4. $\sin^{-1}(\sin u) = u$, Same basic argument as in 3 above, final conclusion $-\frac{\pi}{2} \le u \le \frac{\pi}{2}$.

II on to NEXT Material.

II STudents should take extensive notes !!

III Prove $\sin(\cos^{-1} u) = \sqrt{1-u^2}$

$\left\{ \begin{array}{l} 0 \le x \le \pi \\ |u| \le 1 \end{array} \right.$

Let $x = \cos^{-1} u \Rightarrow \cos x = u$

ORiginal Problem becomes:

$\sin x = \sqrt{1-u^2}$

Take cos of Both sides
$\cos(\cos^{-1} u) = u$ because u has been defined

Remember

Pop it method

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\sin x = \pm\sqrt{1-\cos^2 x}$ Since $u = \cos x$

$u^2 = \cos^2 x$

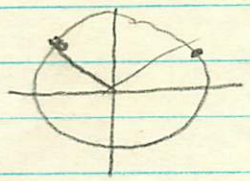
$\sin x = \pm\sqrt{1-u^2}$ Since $x = \cos^{-1} u$

$*$ $\sin(\cos^{-1} u) = \pm\sqrt{1-u^2}$.

It doesn't Quite look like original problem, consider: 1) What is Domain of $\cos^{-1} u = x$? $|u| \le 1$
2) what is range of $\cos^{-1} u = x$? $0 \le x \le \pi$.
So what values can $\sin x$ take on?
if $0 \le x \le \pi$ $\sin x > 0$ only Thus. $*$ becomes:

$\sin(\cos^{-1} u) = \sqrt{1-u^2}$. QED.

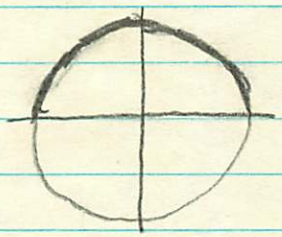
(20) $\cos^{-1}\left(\sin\frac{2\pi}{3}\right)$



$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}$ here range of $\cos^{-1}$ is the key



(21) $\sin\left(\text{arc}\sin\frac{1}{3} + \text{arc}\cos\frac{1}{2}\right)$

let arc $\sin\frac{1}{3} = u$ and arc $\cos\frac{1}{2} = v$

$\sin(u+v)$

$\sin u\cos v + \sin v\cos u$

$\frac{1}{3}\cdot\frac{1}{2} + \sin\left(\cos^{-1}\frac{1}{2}\right)\cdot\cos\left(\sin^{-1}\frac{1}{3}\right)$

$\frac{1}{6} + \left(\sqrt{1-\frac{1}{4}}\right)\left(\sqrt{1-\frac{1}{9}}\right) =$

$\frac{1}{6} + \sqrt{\frac{3}{4}}\cdot\sqrt{\frac{8}{9}}$

$\frac{1}{6} + \frac{\sqrt{3}}{2}\cdot\frac{2\sqrt{2}}{3}$

$\frac{1}{6} + \frac{2\sqrt{6}}{6} = \frac{1+2\sqrt{6}}{6}$

(22) $\cos(\arccos\frac{1}{5} + \arcsin\frac{1}{4})$

$\cos(\cos^{-1}\frac{1}{5}) - \cos(\sin^{-1}\frac{1}{4}) - \sin(\cos^{-1}\frac{1}{5})\sin(\sin^{-1}\frac{1}{4})$

$\frac{1}{5}\cdot\frac{\sqrt{15}}{4} - \frac{1}{4}\cdot\frac{2\sqrt{6}}{5}$

$\frac{\sqrt{15}}{20} - \frac{2\sqrt{6}}{20} = \frac{\sqrt{15}-2\sqrt{6}}{20}$

(23) $\cos(\arcsin\frac{2}{5} + \pi)$

$\cos\sin^{-1}\frac{2}{5}\cos\pi - 0$

$-\cos\sin^{-1}\frac{2}{5}$

$-\sqrt{1-\frac{4}{25}} = -\frac{\sqrt{21}}{5}$

(24) $\sin(\arccos\frac{1}{4} + \pi)$

$\sin(\arccos\frac{1}{4})\cos\pi + 0$

$-\sin\arccos\frac{1}{4}$

$-\frac{\sqrt{15}}{4}$

(25) $\sin(\arccos \frac{3}{5} - \arcsin \frac{3}{5})$

$\sin(\cos^{-1} \frac{3}{5}) \cos(\sin^{-1} \frac{3}{5}) - \frac{3}{5} \cdot \frac{3}{5}$

$\frac{4}{5} \cdot \frac{4}{5} - \frac{3}{5} \cdot \frac{3}{5}$

$\frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

(26) $\cos(\arcsin \frac{3}{5} - \arccos \frac{3}{5})$

$\cos(\sin^{-1} \frac{3}{5}) \cdot \frac{3}{5} + \frac{3}{5} \cdot \sin(\cos^{-1} \frac{3}{5})$

$2(\frac{4}{5} \cdot \frac{3}{5}) = \frac{24}{25}$

(27) $\cos(\arcsin \frac{4}{5} - \frac{\pi}{2})$

$0 + \frac{4}{5} \cdot (1) = \frac{4}{5}$

(28) $\sin(\arccos \frac{4}{5} - \frac{\pi}{2})$

$(\sin \cos^{-1} \frac{4}{5}) \cdot 0 - 1 \cdot \frac{4}{5} = -\frac{4}{5}$

3rd Lecture

Develope these via arguing fctns.

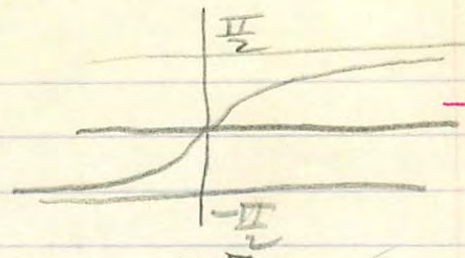
I The other fctns have their own inverse fctns as well.

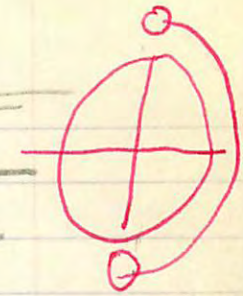
A. $\tan^{-1} x = y$

Notice Domain $x \in \mathbb{R}$.

Strict Range $|y| < \frac{\pi}{2}$

inequality!

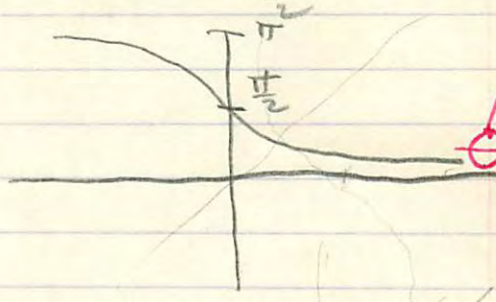


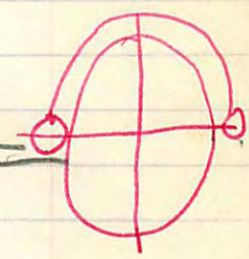


B $\cot^{-1} x = y$

Domain $x \in \mathbb{R}$

Range $0 < y < \pi$





Notice Half Closed Range

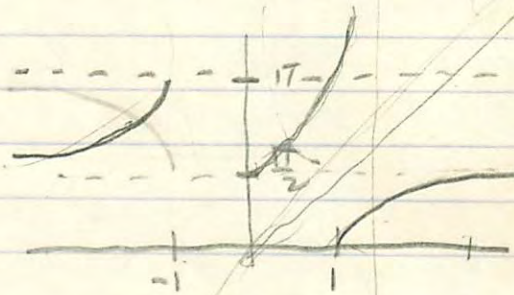
C) $\sec^{-1} x = y$

Domain $|x| \geq 1$

Range $0 \leq y < \frac{\pi}{2}$

or

$\frac{\pi}{2} < y \leq \pi$



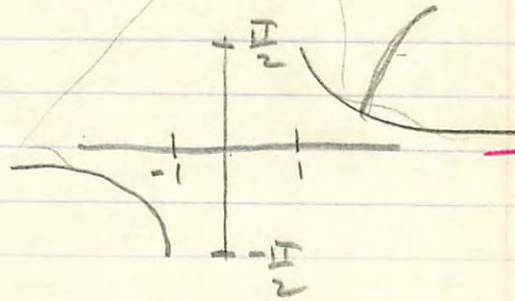
D) $\csc^{-1} x = y$

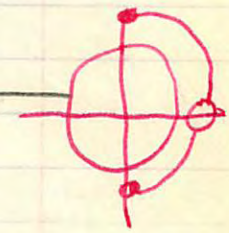
Domain $|x| \geq 1$

Range $\frac{\pi}{2} \geq y > 0$

OR

$0 > y \geq -\frac{\pi}{2}$





II Let students struggle with Routine HW assignment. Catch the finer points after they have learned the Domains + ranges.

HW Pg 147 5-20.

Summary follows

Inverse fcTns

	Domain	Range
$\cos^{-1}$	$-1 \le x \le 1$	$0 \le y \le \pi$
$\sin^{-1}$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$\tan^{-1}$	$\mathbb{R}$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$\cot^{-1}$	$\mathbb{R}$	$0 < y < \pi$
$\sec^{-1}$	$-1 \ge x \ge 1$	$0 \le y \le \pi \quad y \neq \frac{\pi}{2}$
$\csc^{-1}$	$-1 \ge x \ge 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2} \quad y \neq 0$

POOR!

4th lecture.

I Prove $\cos^{-1}x = \sec^{-1}\left(\frac{1}{x}\right)$, $x \neq 0$ Pg 146 Eg 1

Let $y = \cos^{-1}x$ $0 \leq y \leq \pi$ Since $x \neq 0$, $y \neq \frac{\pi}{2}$

$\cos y = x$ $\cos^{-1}x$ is in Domain on Cos. By definition of $\cos^{-1}x$, it is a Real Number, Domain of Cos is $\mathbb{R}$.

Before starting Do answers to Section V on Back of HW sked.

$\sec y = \frac{1}{x}$ Def of Sec fctN.

$y = \sec^{-1}\left(\frac{1}{x}\right)$ True if $\sec y$ is in domain of $\sec^{-1}$. Since $\sec y = \frac{1}{x}$, the Question becomes: "Is $\frac{1}{x}$ in domain of $\sec^{-1}$"?

① Since $x \neq 0$ $\frac{1}{x}$ is defined

② Since $x = \cos y \Rightarrow |x| \leq 1 \Rightarrow \left|\frac{1}{x}\right| \geq 1$

③ Domain of $\sec^{-1}u$ $|u| \geq 1$

④ Since ② and ③ are the same, $\sec y$ is in domain of $\sec^{-1}x$.

$\cos^{-1}x = \sec^{-1}\left(\frac{1}{x}\right)$ Since $y = \cos^{-1}x$ QED.

II Prove $\sin(\tan^{-1}u) = \frac{u}{\sqrt{1+u^2}}$ $u \in \mathbb{R}$

Notice "Let". Must be there!

Let $\tan^{-1}u = y$ We are restricting $y \ni |y| < \frac{\pi}{2}$

$\tan y = u$ Tan is defined under restrictions above, and $u \in \mathbb{R}$.

$\frac{\sin y}{\cos y} = u$ Def of Tan.

$\sin y = u\cos y$

$\sin^2 y = u^2\cos^2 y$

$\sin^2 y = u^2(1-\sin^2 y)$ Since I want $\sin y$ I eliminate $\cos y$.

$\sin^2 y = u^2 - u^2\sin^2 y$

$\sin^2 y + u^2\sin^2 y = u^2$

$\sin^2 y(1+u^2) = u^2$

$\sin^2 y = \frac{u^2}{1+u^2}$

(over)

II cont.

$$\sin^2 Y = \frac{u^2}{1+u^2}$$

$$\sin Y = \pm \frac{u}{\sqrt{1+u^2}}$$

we may have introduced an extraneous root.

Since $Y = \tan^{-1} u$ $\quad -\frac{\pi}{2} < Y < \frac{\pi}{2}$

$0 \le Y < \frac{\pi}{2}$

Sin and Cos are both Positive in 1st Quad. Therefore we would choose the Positive Sq Root,

$-\frac{\pi}{2} < Y \le 0$

Fourth Quad. $\cos \ge 0$
$\sin x \le 0$.
Since $\sin(u) < 0$ the u
$\frac{u}{\sqrt{1+u^2}}$ is also < 0.
So in order to agree we choose Positive Root,

$\frac{\sin u}{\cos u} = \tan u$ $\quad \cos u > 0$ in either case

Therefore sin is the determining factor.
in 1st Quad all is positive.
In 4th Quad. $\sin u < 0$ and so is $\frac{u}{\sqrt{1+u^2}}$

So for agreement on both sides of equal sign, we take only the positive Root.

III Homework. #5

Pg 148 ~~21-30~~. Determine $\sin^{-1} x \stackrel{?}{=} \frac{1}{\csc^{-1} x}$

$\cos(\tan^{-1} u) = ?$

Pg 148 23, 24.

A) $\sin x = \frac{1}{\csc x}$ $\quad x \notin \{k\pi\}$ $\quad$ Yes.

$= \frac{1}{\frac{1}{\sin x}}$

$= \sin x$

B) $\sin^{-1} x = \frac{1}{\csc^{-1} x}$ $\quad$ No $\quad \sin^{-1}(x) = \csc^{-1}\left(\frac{1}{x}\right)$

Let $\sin^{-1} x = y \quad |y| \le \frac{\pi}{2}$.

$\sin y = x$

$\csc y = \frac{1}{x} \qquad x \neq 0 \Rightarrow y \neq 0$

$y = \csc^{-1}\left(\frac{1}{x}\right)$

C) $\cos(\tan^{-1} u) = ?$

Let $y = \tan^{-1} u \quad |y| < \frac{\pi}{2}, \quad u \in \mathbb{R}$.

$\tan y = u$

$\sin y = u \cos y$

$\sin^2 y = u^2 \cos^2 y$

$1 - \cos^2 y = u^2 \cos^2 y$

$1 = u^2 \cos^2 y + \cos^2 y$

$= \cos^2 y (1 + u^2)$

$\frac{1}{1+u^2} = \cos^2 y$

$\cos y = \frac{\pm 1}{\sqrt{1+u^2}}$

Notice y is in I & IV Quad.
& cos of element in 1st & 4th Quad ≥ 0.
We drop $\pm$ sign. So

$\cos(\tan^{-1} u) = \frac{1}{\sqrt{1+u^2}}$

D Pg 148 23, 24

(23) $\sin\left(\sin^{-1}\frac{5}{13} + \tan^{-1}\frac{1}{2}\right) =$

$\sin\sin^{-1}\frac{5}{13}\cos\tan^{-1}\frac{1}{2} + \sin\tan^{-1}\frac{1}{2}\cos\sin^{-1}\frac{5}{13}$

$\frac{5}{13}\cdot\frac{1}{\sqrt{1+\left(\frac{1}{2}\right)^2}} + \frac{\frac{1}{2}}{\sqrt{1+\frac{1}{4}}}\sqrt{1-\left(\frac{5}{13}\right)^2}$

$\frac{5}{13}\cdot\frac{1}{\frac{\sqrt{4+1}}{4}} + \frac{\frac{1}{2}}{\frac{\sqrt{5}}{2}}\cdot\frac{\sqrt{169-25}}{13}$

$\frac{5}{13}\cdot\frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}}\cdot\frac{12}{13}$

$\frac{2\sqrt{5}}{13} + \frac{12\sqrt{5}}{65}$

$\frac{10\sqrt{5}+12\sqrt{5}}{65} = \frac{22\sqrt{5}}{65}$

(24) $\cos\left(\tan^{-1}\frac{12}{13} + \sin^{-1}\frac{1}{2}\right)$

$\cos\tan^{-1}\frac{12}{13}\cos\sin^{-1}\frac{1}{2} - \sin\tan^{-1}\frac{12}{13}\sin\sin^{-1}\frac{1}{2}$

$\frac{1}{\frac{\sqrt{169+144}}{13}}\cdot\sqrt{1-\left(\frac{1}{2}\right)^2} - \frac{\frac{12}{13}}{\frac{\sqrt{313}}{13}}\cdot\frac{1}{2}$

$\frac{13}{\sqrt{313}}\cdot\frac{\sqrt{3}}{2} - \frac{12}{\sqrt{313}}\cdot\frac{1}{2}$

$\frac{13\sqrt{3}-12}{2\sqrt{313}} = \frac{13\sqrt{939}-12\sqrt{313}}{626}$

(7) $\tan^{-1}(1)$

$\frac{\pi}{4}$

(8) $\sec^{-1} 2$

$\cos^{-1}\left(\frac{1}{2}\right)$

$\frac{\pi}{3}$

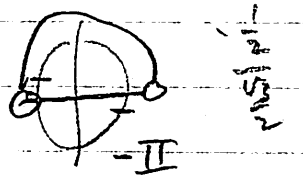
(9) $\sec^{-1}\sqrt{2}$

$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\pi}{4}$

(10) $\cot^{-1}(-\sqrt{3})$

$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$



$-\frac{\pi}{6}$

(11) $\csc^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

$\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

$-\frac{\pi}{3}$

(12) $\tan^{-1} 0$

0

(13) $\csc^{-1} 2$

$\sin^{-1}\frac{1}{2}$

$\frac{\pi}{6}$

(14) $\csc^{-1}(-\sqrt{2})$

$\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right)$

$-\frac{\pi}{4}$

(15) $\tan(\tan^{-1} 1)$

$\tan\left(\frac{\pi}{4}\right)$

1

(16) $\sec\left(\sec^{-1}\frac{2}{\sqrt{3}}\right)$

$\frac{2}{\sqrt{3}}$

(17) $\sec(\sec^{-1} 1)$

1

(18) $\cot\left[\cot^{-1}(-1)\right]$

-1

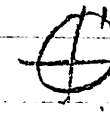
(19) $\tan\left(\cot^{-1}\frac{1}{\sqrt{3}}\right)$

$\tan\left(\frac{\pi}{3}\right)$ $\sqrt{3}$

(20) $\cot\left(\tan^{-1}(-\sqrt{3})\right]$

$\frac{\sqrt{3}}{2}$

$\frac{1}{2}$



$\cot\left(\frac{\pi}{3}\right)$ $\frac{\sqrt{3}}{3}$

(21) $\sin\left(\csc^{-1}\frac{2}{\sqrt{3}}\right)$

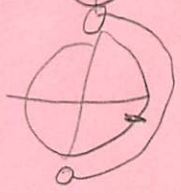
$\frac{\sqrt{3}}{2}$

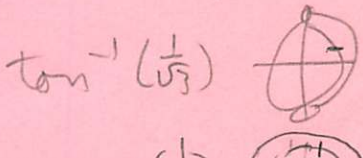
(22)

	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
$\tan^{-1}x$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\cot^{-1}x$	$\frac{5\pi}{6}$	$\frac{3\pi}{4}$	$\frac{2\pi}{3}$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$
$\tan^{-1}x+\cot^{-1}x$	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$
$\tan^{-1}\frac{1}{x}$	$-\frac{\pi}{6}$	$-\frac{\pi}{4}$	$-\frac{\pi}{3}$	DNE	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$
$\tan^{-1}x+\tan^{-1}\frac{1}{x}$	$-\frac{\pi}{2}$	$-\frac{\pi}{2}$	$-\frac{\pi}{2}$	DNE	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$

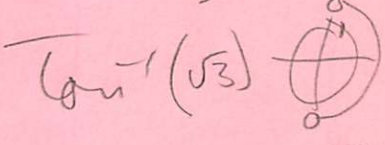
$\tan^{-1}-\sqrt{3}$

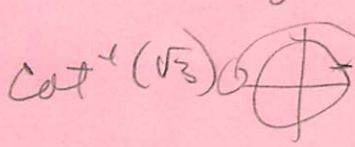
$\cot^{-1}-\sqrt{3}$

$\tan^{-1}\frac{-1}{\sqrt{3}}$ 

$\tan^{-1}(\frac{1}{\sqrt{3}})$ 

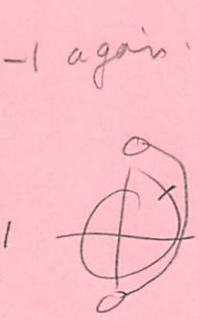
$\cot^{-1}(\frac{1}{\sqrt{3}})$

$\tan^{-1}(\sqrt{3})$ 

$\cot^{-1}(\sqrt{3})$ 

$\tan^{-1}-1$

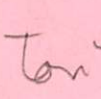
$\cot^{-1}-1$

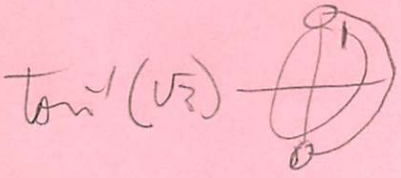
$\tan^{-1}-1$ again 

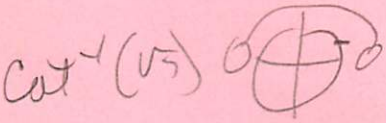
$\tan^{-1}1$

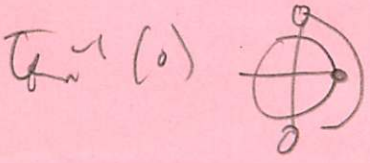
$\cot^{-1}(1)$

$\cot^{-1}(0)$

$\tan^{-1}(-\sqrt{3})$ 

$\tan^{-1}(\sqrt{3})$ 

$\cot^{-1}(\sqrt{3})$ 

$\tan^{-1}(0)$ 

23 (cont)

$\cos^2 y + 4\cos^2 y = 4$

$\cos^2 y (1+4) = 4$

$\cos^2 y = \frac{4}{5}$

$\cos y = \frac{2}{\sqrt{5}}$

$\cos\left(\sin^{-1}\frac{5}{13}\right)$:

$\sin^{-1}\frac{5}{13} = y$

$\sin y = \frac{5}{13}$

$\cos^2 y + \frac{25}{169} = 1$

$\cos^2 y = \frac{144}{169}$

$\cos y = \frac{12}{13}$

$\sin\left(\tan^{-1}\frac{1}{2}\right)$

$\sin(\tan^{-1} u) = \frac{u}{\sqrt{1+u^2}}$

$\frac{\frac{1}{2}}{\sqrt{1+\frac{1}{4}}} = \frac{\frac{1}{2}}{\sqrt{\frac{5}{4}}} = \frac{\frac{1}{2}}{\frac{\sqrt{5}}{2}} = \frac{2}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\frac{5}{13}\cdot\frac{2\sqrt{5}}{5} + \frac{12}{13}\cdot\frac{\sqrt{5}}{5} = \frac{10\sqrt{5}+12\sqrt{5}}{65} = \frac{22\sqrt{5}}{65}$

Eg 2 Pg 146

Redone on 4th lecture (above)

$\sin(\tan^{-1} u) = \frac{u}{\sqrt{1+u^2}}$ $\quad u \in \mathbb{R} \quad |y| < \frac{\pi}{2}$

let $y = \tan^{-1} u$

$\tan y = u$

$\frac{\sin y}{\cos y} = u$

$\sin y = u \cos y$

$\sin^2 y = u^2 \cos^2 y$

$\sin^2 y = u^2(1 - \sin^2 y)$

$\sin^2 y = u^2 - u^2 \sin^2 y$

$\sin^2 y + u^2 \sin^2 y = u^2$

$\sin^2 y (1+u^2) = u^2$

$\sin^2 y = \frac{u^2}{1+u^2}$

$\sin y = \pm \frac{u}{\sqrt{1+u^2}}$

$\sin(\tan^{-1} u) = \frac{u}{\sqrt{1+u^2}}$

Since $y = \tan^{-1} u$ $|y| < \frac{\pi}{2}$ sin and tan have the same sign thus we pick positive sq root

---

$\cos(\tan^{-1} u) =$

let $\tan^{-1} u = y$

$\tan y = u$

$\sin y = u \cos y$

$\sin^2 y = u^2 \cos^2 y$

$1 - \cos^2 y = u^2 \cos^2 y$

$1 = u^2 \cos^2 y + \cos^2 y$

$1 = (u^2+1) \cos^2 y$

$\frac{1}{u^2+1} = \cos^2 y$

$\frac{1}{\sqrt{u^2+1}} = y,$

① $\tan y = 1 \quad y \in \left\{\frac{\pi}{4} + \pi k\right\}$

② $\sec y = \sqrt{2} \quad y \in \begin{cases} \frac{\pi}{4} + 2k\pi \\ \frac{7\pi}{4} + 2k\pi \end{cases}$

③ $\cot y = -1 \quad y \in \left\{\frac{3\pi}{4} + k\pi\right\}$

④ $\csc y = -\sqrt{2} \quad y \in \begin{cases} \frac{5\pi}{4} + 2k\pi \\ \frac{2\pi}{4} + 2k\pi \end{cases}$

⑤ $\arctan(-1) = y$

$-1 = \tan y \quad y = \frac{3\pi}{4}, \frac{7\pi}{4}$

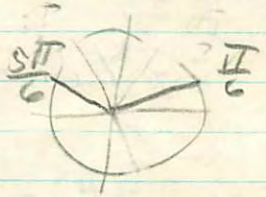
$y = \frac{7\pi}{4} = -\frac{\pi}{4}$ Range is $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

⑥ $\operatorname{arcsec}\sqrt{2}$

$\sqrt{2} = \sec y$

Range $0 \le y \le \pi$ $\therefore$ $y = \frac{\pi}{4}$

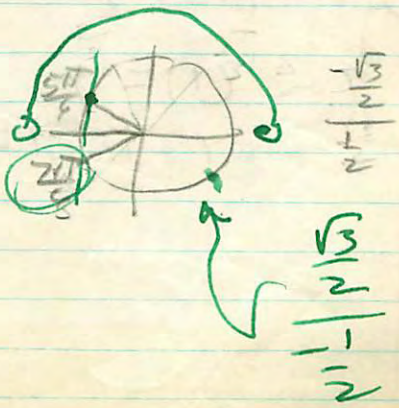
⑦ $\operatorname{arccsc} 2 = y$



$2 = \csc y$

Range $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ $\therefore$ $y = \frac{\pi}{6}$

⑧ $\operatorname{arccot}(-\sqrt{3}) = y$



$-\sqrt{3} = \cot y$

Range $0 \le y \le \pi$ $\therefore$ $y = \frac{5\pi}{6}$

(9) $\cot^{-1}(1) = Y$

$1 = \cot Y$

$Y = \frac{\pi}{4}$

(10) $\csc^{-1} 1 = Y$

$Y = \frac{\pi}{2}$

(11) $\sec^{-1} \frac{2}{\sqrt{3}}$

$\frac{2}{\sqrt{3}} = \sec Y$

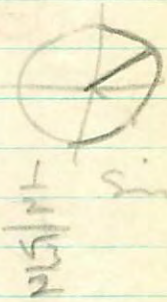
$\frac{\sqrt{3}}{2} = \cos Y$

$Y = \frac{\pi}{6}$

(12) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$\tan Y = \frac{\sqrt{3}}{3}$

$Y = \frac{\pi}{6}$



(13) $\cos(\arctan 1)$

$\arctan 1 = Y$

$1 = \tan Y$

$Y = \frac{\pi}{4}$

$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

(14) $\tan(\arccos \frac{1}{2})$

$\arccos \frac{1}{2} = Y$

$\cos Y = \frac{1}{2}$

$Y = \frac{\pi}{3}$

$\tan \frac{\pi}{3} \quad \frac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

(15) $\sec(\cos^{-1} \frac{2}{5})$

$\cos^{-1} \frac{2}{5} = Y$

$\frac{2}{5} = \cos Y \quad \therefore \sec Y = \frac{5}{2}$

(16) $\csc(\sin^{-1} \frac{2}{5})$

$\sin^{-1} \frac{2}{5} = Y$

$\frac{2}{5} = \sin Y \quad \therefore \csc Y = \frac{5}{2}$

(17) $\tan^{-1}(\cot \frac{\pi}{4}) = Y$

$\tan^{-1}(1) = Y$

$\tan Y = 1 \quad Y = \frac{\pi}{4}$

(18) $\sec^{-1}(\csc \frac{\pi}{3}) = Y$

$\sec^{-1}(\frac{2\sqrt{3}}{3}) = Y$

$\frac{\sqrt{3}}{2} = \cos Y$

$Y = \frac{\pi}{6}$

(19) $\sec^{-1}(\tan\frac{\pi}{4}) = Y$

$\sec^{-1}(1) = Y$

$\cos Y = 1$

$Y = \frac{\pi}{2}$

(20) $\cot^{-1}(2\sin^2\frac{2\pi}{3})$

$\cot^{-1}(\sqrt{3}) = Y$

$\sqrt{3} = \cot Y$

$\frac{\sqrt{3}}{2}$ cos

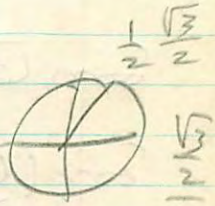
$\frac{1}{2}$ sin

$Y = \frac{\pi}{6}$

(21) $\tan(\tan^{-1}\frac{2}{5} + \frac{\pi}{3})$

$$\frac{\tan(\tan^{-1}\frac{2}{5}) + \tan\frac{\pi}{3}}{1 - \tan(\tan^{-1}\frac{2}{5})\tan\frac{\pi}{3}} = \frac{\frac{2}{5} + \sqrt{3}}{1 - \sqrt{3}\cdot\frac{2}{5}} = \frac{\frac{2+5\sqrt{3}}{5}}{\frac{5-2\sqrt{3}}{5}} = \frac{2+5\sqrt{3}}{5-2\sqrt{3}}$$

$$\frac{2+5\sqrt{3}}{(5-2\sqrt{3})}\frac{(5+2\sqrt{3})}{(5+2\sqrt{3})} = \frac{10+29\sqrt{3}+30}{25-12} = \frac{40+29\sqrt{3}}{13}$$



(22) $\tan(\tan^{-1}\frac{2}{3} + \frac{\pi}{4})$

$$\frac{\tan(\tan^{-1}\frac{2}{3}) + \tan\frac{\pi}{4}}{1 - \tan(\tan^{-1}\frac{2}{3})\tan\frac{\pi}{4}} = \frac{\frac{2}{3} + 1}{1 - \frac{2}{3}} = \frac{\frac{5}{3}}{\frac{1}{3}} = 5$$

(23) $\sin(\sin^{-1}\frac{5}{13} + \tan^{-1}\frac{1}{2})$

$\sin(\sin^{-1}\frac{5}{13})\cos(\tan^{-1}\frac{1}{2}) + \cos(\sin^{-1}\frac{5}{13})\sin(\tan^{-1}\frac{1}{2})$

Let $\tan^{-1}\frac{1}{2} = Y$

$\frac{1}{2} = \tan Y = \frac{\sin Y}{\cos Y}$

$\cos Y = 2\sin Y$

$\cos^2 Y = 4\sin^2 Y$

$\cos^2 Y = 4(1 - \cos^2 Y)$

$\cos^2 Y = 4 - 4\cos^2 Y$

$\tan^{-1}\frac{1}{2} = Y$

$\tan Y = \frac{1}{2}$

21 $\tan\left(\tan^{-1}\frac{2}{5}+\frac{\pi}{3}\right)$

$$\frac{\sin\tan^{-1}\frac{2}{5}\cos\frac{\pi}{3}+\sin\frac{\pi}{3}\cos\tan^{-1}\frac{2}{5}}{\cos\tan^{-1}\frac{2}{5}\cos\frac{\pi}{3}-\sin\tan^{-1}\frac{2}{5}\sin\frac{\pi}{3}}$$

$$\sin\tan^{-1}u=\frac{u}{\sqrt{1+u^2}}$$

$$\sin\left(\tan^{-1}\frac{2}{5}\right)=\frac{\frac{2}{5}}{\sqrt{1+\frac{4}{25}}}=\frac{\frac{2}{5}}{\frac{\sqrt{29}}{5}}=\frac{2}{\sqrt{29}}\quad\therefore$$

$$\cos\left(\tan^{-1}\frac{2}{5}\right)=\frac{5}{\sqrt{29}}$$

Quite est dite

$$\frac{\frac{2}{\sqrt{29}}\cdot\frac{1}{2}+\frac{\sqrt{3}}{2}\frac{5}{\sqrt{29}}}{\frac{5}{\sqrt{29}}\cdot\frac{1}{2}-\frac{2}{\sqrt{29}}\cdot\frac{\sqrt{3}}{2}}$$

$$\frac{\frac{2+5\sqrt{3}}{2\sqrt{29}}}{\frac{5-2\sqrt{3}}{2\sqrt{29}}}\qquad\frac{5+2\sqrt{3}}{5+2\sqrt{3}}=$$

24. $\cos\left(\tan^{-1}\frac{12}{13} + \sin^{-1}\frac{1}{2}\right)$

$\cos\left(\tan^{-1}\frac{12}{13}\right)\cos\left(\sin^{-1}\frac{1}{2}\right) - \sin\left(\tan^{-1}\frac{12}{13}\right)\sin\left(\sin^{-1}\frac{1}{2}\right)$

$\frac{1}{\sqrt{\frac{144}{169}+1}} \cdot \frac{\sqrt{3}}{2} - \frac{\frac{12}{13}}{\sqrt{\frac{313}{169}}} \cdot \frac{1}{2}$

$\frac{1}{\sqrt{\frac{313}{169}}} - \frac{6}{\sqrt{313}}$

$\frac{\sqrt{169}}{\sqrt{313}} \qquad \frac{13\sqrt{3}}{2\sqrt{313}} - \frac{12}{2\sqrt{313}} = \frac{13\sqrt{3}-12}{2\sqrt{313}} =$

$\frac{13}{2\sqrt{313}} \qquad \frac{13\sqrt{939}-12\sqrt{313}}{626}$

$\cos\tan^{-1}u = \frac{1}{\sqrt{1+u^2}}$

$\cos\left(\sin^{-1}\frac{1}{2}\right)$

$\sin\gamma = \frac{1}{2}$

$\frac{1}{4} + \cos^2\gamma = 1$

$\cos^2\gamma = \frac{3}{4}$

$\cos\gamma = \frac{\sqrt{3}}{2}$

---

25. $\sin\left(2\cos^{-1}\frac{3}{5}\right) = 2\sin\left(\cos^{-1}\frac{3}{5}\right)\cos\left(\cos^{-1}\frac{3}{5}\right)$

$2\left(\sqrt{1-\frac{9}{25}}\right)\left(\frac{3}{5}\right)$

$2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$\frac{24}{25}$

27 $\sin\left(\frac{1}{2}\cos^{-1}\frac{5}{13}\right)$ use $\frac{1}{2}$ θ form pg 8 #7

$\sin\frac{x}{2} = \sqrt{\frac{1}{2}(1-\cos x)}$

$\sin\left(\frac{1}{2}\cos^{-1}\frac{5}{13}\right) = \sqrt{\frac{1}{2}\left(1-\cos\cos^{-1}\frac{5}{13}\right)}$

$= \sqrt{\frac{1}{2}\left(1-\frac{5}{13}\right)}$

$= \sqrt{\frac{1}{2}\left(\frac{13-5}{13}\right)}$

$= \sqrt{\frac{4}{13}} = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$

$= \frac{\sqrt{52}}{13} = \frac{2\sqrt{13}}{13}$

#29 $\sin^{-1} u = \tan^{-1} \frac{u}{\sqrt{1-u^2}}$ $|u|<1$

Let $\sin^{-1} u = y$ $|y| \le \frac{\pi}{2}$

$\sin y = u$

$\sin^2 y = u^2$ $\sin^2 y + \cos^2 y = 1 \therefore$ $\cos^2 y = 1-u^2$

$\frac{\sin^2 y}{\cos^2 y} = \frac{u^2}{\cos^2 y}$ $y \neq \frac{\pi}{2} \therefore |y| < \frac{\pi}{2}$

$\tan^2 y = \frac{u^2}{1-u^2}$

$\tan y = \frac{\pm u}{\sqrt{1-u^2}}$ $y \in 1^{ST}$ & 4th Quad. IN 1ST Quad all >0. In 4th Quad $\tan y < 0$ and so should u $\therefore$ No need for additional Neg.

$\tan y = \frac{u}{\sqrt{1-u^2}}$

$y = \tan^{-1} \frac{u}{\sqrt{1-u^2}}$ $\tan(\tan^{-1} y) = y$ if $|y| < \frac{\pi}{2}$

$\sin^{-1} u = \tan^{-1} \frac{u}{\sqrt{1-u^2}}$

#30 $\tan^{-1} x_1 + \tan^{-1} x_2 = \tan^{-1}\left(\frac{x_1 + x_2}{1 - x_1 x_2}\right)$ $x_1 x_2 \neq 1$

operate w/ tan

$\tan\left(\tan^{-1} x_1 + \tan^{-1} x_2\right) = \tan\left(\tan^{-1} \frac{x_1 + x_2}{1 - x_1 x_2}\right)$

$\frac{\tan\tan x_1 + \tan\tan^{-1} x_2}{1 - \tan\tan^{-1} x_1 \tan\tan^{-1} x_2} = \frac{x_1 + x_2}{1 - x_1 x_2}$

$\frac{x_1 + x_2}{1 - x_1 x_2} =$

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31 $\sin(2\tan^{-1}x) = \frac{2x}{1+x^2}$

$2\sin(\tan^{-1}x)\cos\tan^{-1}x = 2\left(\frac{x}{\sqrt{1+x^2}} \cdot \frac{1}{\sqrt{1+x^2}}\right)$

$= \frac{2x}{1+x^2}$

These are important probms, used to Simplify Pg 157 #19

Back of HW Skid #W#7

32 $\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right) \quad x \neq 0$

Let $\cot^{-1}x = a$

$\cot a = x$

$\tan a = \frac{1}{x}$ Since $x \in R, x \neq 0$ $\frac{1}{x}$ is defined and

$a = \tan^{-1}\left(\frac{1}{x}\right)$ also is $\in R$. (Domain $\tan^{-1}$).

$\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$

(33) $\tan^{-1}x = \frac{\pi}{2} - \cot^{-1}x$

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

$\tan^{-1}x + \tan^{-1}\frac{1}{x} = \frac{\pi}{2}$ use #32 above.

Operate w/ cos.

$\cos\tan^{-1}x \cos\tan^{-1}\left(\frac{1}{x}\right) - \sin\tan^{-1}x \sin\tan^{-1}\frac{1}{x} = 0$

$$\left(\frac{1}{\sqrt{1+x^2}}\right)\frac{1}{\sqrt{1+\frac{1}{x^2}}} - \frac{x}{\sqrt{1+x^2}}\,\frac{\frac{1}{x}}{\sqrt{1+\frac{1}{x^2}}} = 0$$

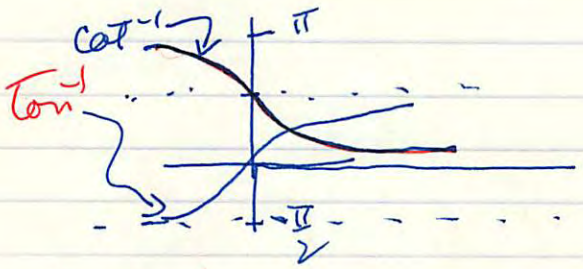
$$\frac{1}{\sqrt{1+x^2}} \cdot \frac{1}{\frac{\sqrt{x^2+1}}{x}} - \frac{x}{\sqrt{1+x^2}} \cdot \frac{\frac{1}{x}}{\frac{\sqrt{x^2+1}}{x}} = 0$$

$$\frac{x}{1+x^2} - \frac{x}{1+x^2} = 0$$

$$0 = 0$$

Extra credit. True Pg 119

$\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right) + \pi \quad x < 0$



consider

Both versions shown on PIMC paper after freedom slip handed out.

If $x < 0$, $\frac{1}{x} < 0$ so $\tan^{-1}\left(\frac{1}{x}\right)$ is an arc length from $-\frac{\pi}{2} \to 0$

This is NOT in the proper interval for $\cot^{-1}$. The "negative portion" of $\tan^{-1}x$ is from $-\frac{\pi}{2} \to 0$ needs to be added to π to bring it into the interval $\frac{\pi}{2} \to \pi$

(34) $\sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$

$x = \sin\frac{\pi}{2}\cos(\cos^{-1}x) - 0$

$x = x$

(35) $\sin(2\cos^{-1}x) = 2\sin\cos^{-1}x\cos\cos^{-1}x =$

$2x\sqrt{1-x^2}$

(36) $\sin\left(\frac{1}{2}\cos^{-1}x\right) = \pm\sqrt{\frac{1}{2}(1-\cos(\cos^{-1}x)} =$

$\pm\sqrt{\frac{1}{2}(1-x)} = \pm\frac{\sqrt{2(1-x)}}{2}$

(37) $\cos\left(\frac{1}{2}\sin^{-1}x\right) = \pm\sqrt{\frac{1}{2}(1+\cos(\sin^{-1}x)} =$

$\pm\sqrt{\frac{1}{2}(1+\sqrt{1-x^2})}$

(38) $\cos(2\sin^{-1}x) = (\cos(\sin^{-1}x))^2 - (\sin\sin^{-1}x)^2$

$1-x^2 - x^2 = 1-2x^2$

(39) $\sin(2\sin^{-1}x) = 2\sin\sin^{-1}x\cos\sin^{-1}x$

$2x\sqrt{1-x^2}$

(40) $\cos(2\cos^{-1}x) = (\cos(\cos^{-1}x))^2 - (\sin(\cos^{-1}x))^2$

$x^2 - (1-x^2) = 2x^2-1$

#1 & #3 15 min 1st Quiz Ch 1985 ~~Actual Graph of fctn extra credit~~ st. each. Required

Trig Quiz.

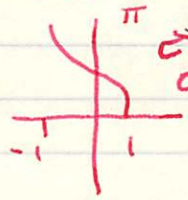
I State Domain and Range of the following functions. Sketch. 4 pts ea. Supply other 5.

Demo:

a) $f(x) = \cos^{-1} x$

Dom $|x| \le 1$

Range $0 \le f(x) \le \pi$



~~extra credit.~~

b) $g(x) = \sin^{-1} x$

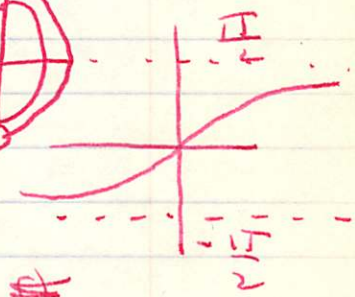
$|x| \le 1$

$|g(x)| \le \frac{\pi}{2}$

c) $h(x) = \tan^{-1} x$

$x \in \mathbb{R}$

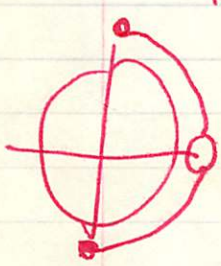
$|\tan^{-1} x| < \frac{\pi}{2}$

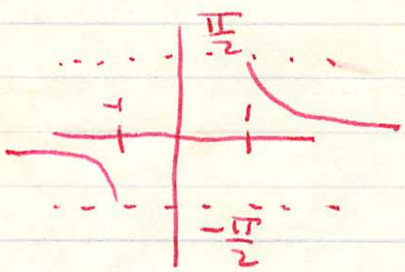


d) $j(x) = \csc^{-1} x$

Dom $|x| \ge 1$

Range $|j(x)| \le \frac{\pi}{2}$ $j(x) \ne 0$

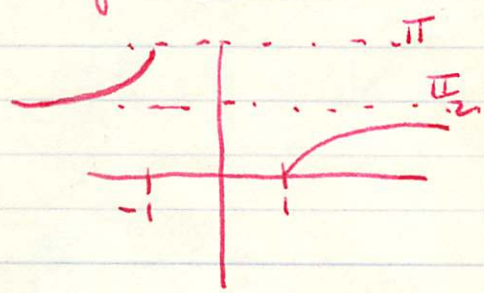




e) $K(x) = \sec^{-1} x$

Dom $|x| \ge 1$

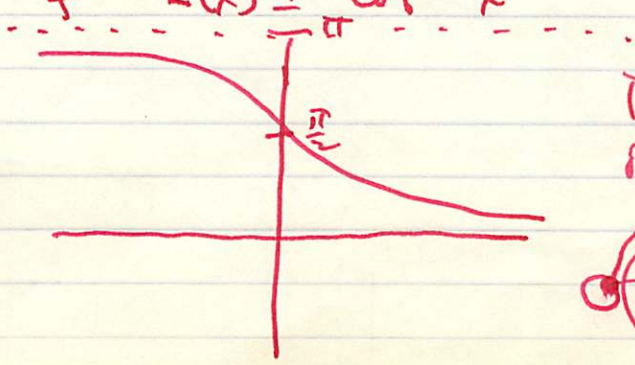
Range $0 \le K(x) \le \pi$ $K(x) \ne \frac{\pi}{2}$



1981 I had students draw circles which represent range ie

etc.

f $L(x) = \cot^{-1} x$



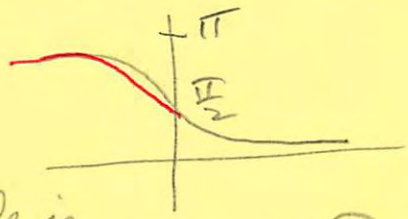
D_L $x \in \mathbb{R}$

R_L $0 < L(x) < \pi$

IV

c) WTP $\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right) + \pi \quad x<0$

Let $\cot^{-1}x = b \quad x<0$, given $\frac{\pi}{2} < b < \pi$



$\cot(\cot^{-1}x) = \cot b$

$x = \cot b$

Left side is x because $D_{\cot^{-1}x} \in \mathbb{R}$, $R_{\cot} \in \mathbb{R}$ So no restrictions.

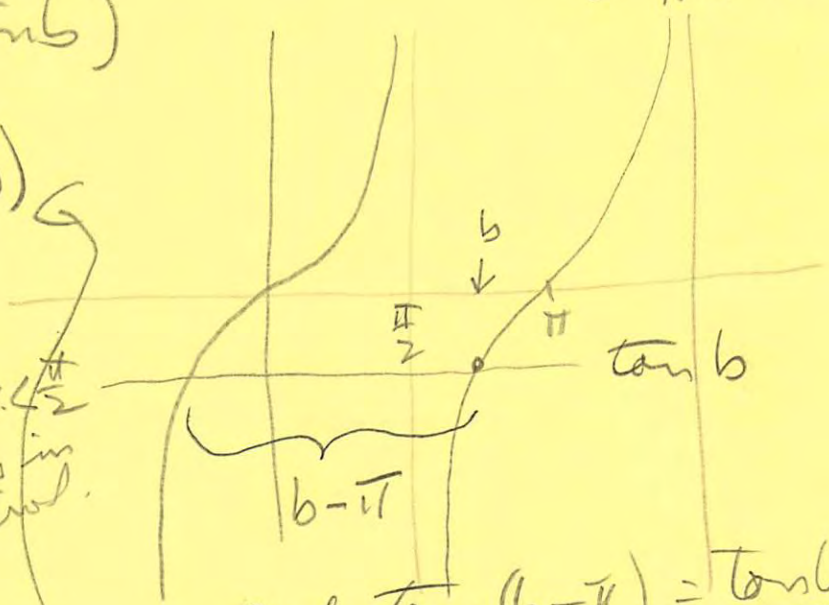
$\frac{1}{x} = \tan b \quad x \neq 0$ because $x<0$ $\frac{1}{\tan b} = \cot b$ if $b \neq \frac{k\pi}{2}$ and b is between $\frac{\pi}{2}$ and π Not inclusive

$\tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(\tan b)$

$\tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(\tan(b-\pi))$

$\tan^{-1}\frac{1}{x} = b - \pi$

because $R_{\tan^{-1}}: -\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{2}$ in and $b-\pi$ is in same interval.



and $\tan(b-\pi) = \frac{\tan b - \tan\pi}{1 - \tan b \tan\pi}$

$= \frac{\tan b - 0}{1 - 0}$

$= \tan b.$

So Sub $\tan(b-\pi)$ in for $\tan b$

$\tan^{-1}\frac{1}{x} = \cot x - \pi$

$\cot^{-1}x = \tan^{-1}\frac{1}{x} + \pi$

QED.

additional Problems - U PROVE-ITs

$\tan(\tan^{-1}u) = u \qquad u \in \mathbb{R}$

$\tan^{-1}(\tan u) = u \qquad |u| < \frac{\pi}{2}$

$\csc(\csc^{-1}u) = u \qquad |u| \geq 1$

$\csc^{-1}(\csc u) = u \qquad |u| \leq \frac{\pi}{2} \quad u \neq 0$

$\sec(\sec^{-1}u) = u \qquad |u| \geq 1$

$\sec^{-1}(\sec u) = u \qquad 0 \leq u \leq \pi \quad u \neq \frac{\pi}{2}$

$\cot(\cot^{-1}u) = u \qquad u \in \mathbb{R}$

$\cot^{-1}(\cot u) = u \qquad 0 < u < \pi$

$\tan^{-1}u \overset{?}{=} \frac{\sin^{-1}u}{\cos^{-1}u}$ ← the Gambler's Problm.

let $x = \tan^{-1}u$ then $|x| < \frac{\pi}{2}$

let $y = \sin^{-1}u$ then $|y| \leq \frac{\pi}{2}$

let $z = \cos^{-1}u \qquad u \neq \frac{\pi}{2}$

$z = 0 \leq u \leq \pi$ notice we do not include $\frac{\pi}{2}$ since $\cos^{-1}u$ is in denominator

If $x = \frac{y}{z}$ then abv stmt is true,

Notice that the "Ranges" are different.". START is Not true " for elements, u, which are defined for those fctns. Similar eg: $\frac{(x+1)(x+2)}{x+2} = f(x)$

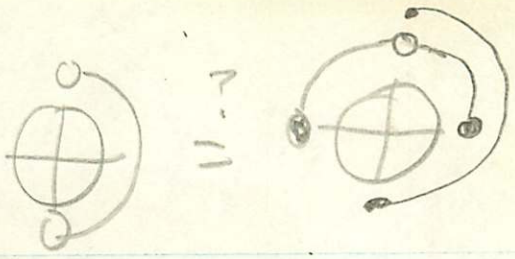
$g(x) = x+1$. $f(x) \neq g(x)$ Because Domains are different,

also notice $u = \frac{3\pi}{4}$ is not Defined on Right Side

(over)

As a matter of fact, maybe.....

$$\tan^{-1} u = \frac{\sin^{-1} u}{\cos^{-1} u} \qquad 0 \leq u < \frac{\pi}{2}$$



$\tan^{-1} u \stackrel{?}{=} \dfrac{\sin^{-1} u}{\cos^{-1} u}$

Range on the left: $|\tan^{-1} a| < \frac{\pi}{2}$

Range in Numerator on Right $|\sin^{-1} u| \leq \frac{\pi}{2}$

Range in Denominator on Right $0 \leq \cos^{-1} \leq \pi$
(Since it is a denominator) $\cos^{-1} u \neq \frac{\pi}{2}$

$\therefore$ Range on Right is 1ST Quad, but not $\frac{\pi}{2}$.

1) $\sin x - 1 = 0$

$\sin x = 1$

$x = \frac{\pi}{2}$

2) $\cos x + 1 = 0$

$\cos x = -1$

$x = \pi$

3) $\sqrt{2}\sin x - 1 = 0$

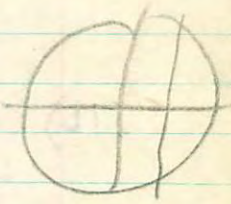
$\sin x = \frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

4) $2\cos x + \sqrt{3} = 0$

$\cos x = -\frac{\sqrt{3}}{2}$

$x = \frac{5\pi}{6}, \frac{7\pi}{6}$



5) $\sin 2x = \sin x$

$2\sin x \cos x = \sin x$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$\sin x = 0$

$\left\{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$

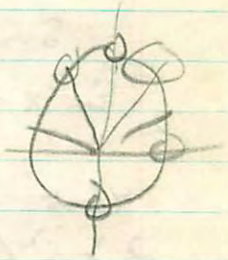
6) $\sin 2x = \cos x$

$2\sin x \cos x = \cos x$

$2\sin x = 1$

$\sin x = \frac{1}{2}$ $x \in \left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

$\cos x = 0$



7) $\sin^2 x = 0$

$\sin x = 0$ $\{0, \pi\}$

8) $4\sin^2 x - 1 = 0$

$(2\sin x + 1)(2\sin x - 1)$

$\sin x = \pm\frac{1}{2}$ $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

9) $2\cos^2 x + 3\cos x + 1 = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$\cos x = -\frac{1}{2}$ $\cos x = -1$

$\left\{\frac{2\pi}{3}, \frac{4\pi}{3}, \pi\right\}$

OK

10) $\sin^2 x - 2\sin x + 1 = 0$

$(\sin x - 1)^2 = 0$

$\sin x = 1$ $\left\{\frac{\pi}{2}\right\}$

P153

11) $\cos 2x + \sin x - 1 = 0$

$\cos^2 x + \sin x - \sin^2 x - 1 = 0$

$-\sin^2 x + \sin x = 1 - \cos^2 x$

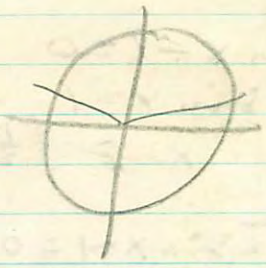
$-\sin^2 x + \sin x = \sin^2 x$

$-2\sin^2 x + \sin x = 0$

$\sin x(-2\sin x + 1) = 0$

$\sin x = 0 \qquad 2\sin x = 1$

$\sin x = \frac{1}{2}$



$0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$

12) $2\cos^2 x = 1 + \cos^2 x$

$\cos^2 x - 1 = 0$

$\cos x = \pm 1 \qquad \{0, \pi\}$

13) $\sin x - \sqrt{3}\cos x = 1$

$\sin x - 1 = \sqrt{3}\cos x$

$\sin^2 x + 2\sin x + 1 = 3\cos^2 x$

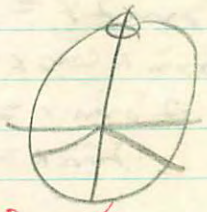
$3(1 - \sin^2 x)$

$4\sin^2 x - 2\sin x - 2 = 3 - 3\sin^2 x$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1)$

$\sin x = -\frac{1}{2} \qquad \sin x = 1$



$\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

Check

$\sin\frac{\pi}{2} - \sqrt{3}\cos\frac{\pi}{2} = 1$

$1 - 0 = 1$

$1 = 1$

$\sin\frac{7\pi}{6} - \sqrt{3}\cos\frac{7\pi}{6} = 1$

$-\frac{1}{2} - \sqrt{3}\cdot\frac{\sqrt{3}}{2} = 1$

$-\frac{1}{2} - \frac{3}{2} = 1$

$-2 = 1$

$\sin\frac{11\pi}{6} - \sqrt{3}\cos\frac{11\pi}{6} = 1$

$-\frac{1}{2} - \sqrt{3}\cdot\frac{\sqrt{3}}{2} = 1$

$-\frac{1}{2} - \frac{3}{2} = 1$

$-2 = 1$

$\sin\frac{2\pi}{6} - \sqrt{3}\cos\frac{2\pi}{6}$

$-\frac{1}{2} - \sqrt{3}\cdot\frac{-\sqrt{3}}{2} = 1$

$-\frac{1}{2} + \frac{3}{2} = 1$

$1 = 1$

14)

$\sin 2x + \cos 2x - 1 = 0$

$2\sin x\cos x - \sin^2 x + \cos^2 x - 1 = 0$

$2\sin x\cos x - \sin^2 x - \sin^2 x = 0$

$2\sin x\cos x - 2\sin^2 x = 0$

$\sin x(\cos x - \sin x) = 0$

$\sin x = 0 \qquad \cos x = \sin x$

$x = 0, \pi \qquad x = \frac{\pi}{4}, \frac{5\pi}{4}$

or: $\sin 2x = 1 - \cos 2x$

$1 - \cos^2 2x = 1 - 2\cos 2x + \cos^2 2x$

$2\cos^2 2x - 2\cos 2x = 0$

$2\cos 2x(\cos 2x - 1) = 0$

$\cos 2x = 0 \qquad \cos 2x = 1$

$2x \in \{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}\} \qquad 2x \in \{0, 2\pi\}$

$x \in \{0, \pi\}$

$\{0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}\}$

$x \in \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, 0, \pi\}$

Now ck is Necessary.

15) $-\sqrt{3}\sin x - \cos x = 1$

$-\sqrt{3}\sin x = 1 + \cos x$

$3\sin^2 x = 1 + 2\cos x + \cos^2 x$

$3(1 - \cos^2 x)$

$3 - 3\cos^2 x = 1 + 2\cos x + \cos^2 x$

$4\cos^2 x + 2\cos x - 2$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1)$

$\cos x = \frac{1}{2} \qquad \cos x = -1$

$\{\cancel{\frac{\pi}{3}}, \frac{5\pi}{3}, \pi\}$

check $-\sqrt{3}\cdot\sin\frac{\pi}{3} - \cos\frac{\pi}{3} = 1$

$-\frac{\sqrt{3}\cdot\sqrt{3}}{2} - \frac{1}{2} = 1$

$-\frac{3}{2} - \frac{1}{2} = 1$

$-\sqrt{3}\sin\frac{5\pi}{3} - \cos\frac{5\pi}{3} = 1$

$-\sqrt{3}\cdot -\frac{\sqrt{3}}{2} - \frac{1}{2} = 1$

$\frac{3}{2} - \frac{1}{2} = 1$

$1 = 1$

$\sqrt{3}\sin\pi - \cos\pi = 1$

$0 - (-1) = 1$

$1 = 1$

16. $\sin 2x + \cos x + 2\sin x = -1$

$2\sin x\cos x + \cos x + 2\sin x = -1$

$\cos x(2\sin x + 1) + 2\sin x = -1$

$\cos x(2\sin x + 1) + 2\sin x + 1 = 0$

$(2\sin x + 1)(\cos x + 1) = 0$

$\sin x = -\frac{1}{2} \quad \cos x = -1$

$\left\{\pi, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

17. $\sin\left(x - \frac{\pi}{4}\right) = \cos\left(x + \frac{\pi}{4}\right)$

$\sin x\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos x = \cos x\cos\frac{\pi}{4} - \sin x\sin\frac{\pi}{4}$

$\frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x = \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$

$\sin x - \cos x = \cos x - \sin x$

$2\sin x - 2\cos x = 0$

$\sin x = \cos x$

$\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

18. $\sin x = \cos(2x - \pi)$

$\sin x = \cos 2x\cos\pi + \sin 2x\sin\pi$

$\sin x = -\cos 2x$

$\sin x = -\cos^2 x + \sin^2 x$

$\sin x = -1 + \sin^2 x + \sin^2 x$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1)$

$\sin x = -\frac{1}{2} \quad \sin x = 1$

$\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

#19 $\sin\left(2x+\frac{2\pi}{3}\right)+\sin\left(2x-\frac{2\pi}{3}\right)=\frac{\sqrt{2}}{2}$

$\sin 2x\cos\frac{2\pi}{3}+\sin\frac{2\pi}{3}\cos 2x+\sin 2x\cos\frac{2\pi}{3}-\sin\frac{2\pi}{3}\cos 2x=\frac{\sqrt{2}}{2}$

$-\frac{1}{2}\sin 2x-\frac{1}{2}\sin 2x=\frac{\sqrt{2}}{2}$

$\sin 2x=\frac{-\sqrt{2}}{2}$

NOTICE →

Restrictions are $0\le x<2\pi$

Thus restrictions on $2x$ are $0\le 2x<4\pi$

That is 2 complete revolutions!

So choices of $2x$ are: $\left\{\frac{5\pi}{4},\frac{7\pi}{4},\frac{13\pi}{4},\frac{15\pi}{4}\right\}$

Since we want choices of x rather than $2x$ we divide by 2,

$$x\in\left\{\frac{5\pi}{8},\frac{7\pi}{8},\frac{13\pi}{8},\frac{15\pi}{8}\right\}.$$

Notice x is, indeed, within the original limits

HW Pg 153 17–21

Extra credit 22, 23.

19. $\sin\left(2x+\frac{2\pi}{3}\right)+\sin\left(2x-\frac{2\pi}{3}\right)=\frac{\sqrt{2}}{2}$

$\sin 2x\cos\frac{2\pi}{3}+\sin\frac{2\pi}{3}\cos 2x+\sin 2x\cos\frac{2\pi}{3}-\sin\frac{2\pi}{3}\cos 2x=\frac{\sqrt{2}}{2}$

$-\sin 2x=\frac{\sqrt{2}}{2}$

$\sin 2x=-\frac{\sqrt{2}}{2}$ $\Rightarrow$ $2x=\frac{5\pi}{4},\ \frac{7\pi}{4},\ \frac{13\pi}{4},\ \frac{15\pi}{4}$

$2x=y$

$\sin y=-\frac{\sqrt{2}}{2}$

$x\in\left\{\frac{5\pi}{8},\ \frac{7\pi}{8},\ \frac{13\pi}{8},\ \frac{15\pi}{8}\right\}$

20. $\sin(\pi-2y)=\cos x$

because $0\le x<2\pi$ $\therefore$ $0\le 2x\le 4\pi$

$\sin\pi\cos 2x-\sin 2x\cos\pi=\cos x$

$\sin 2x=\cos x$

$2\sin x\cos x=\cos x$ $\quad\cos x=0$

$2\sin x=1$

$\sin x=\frac{1}{2}$

$\left\{\frac{\pi}{6},\ \frac{\pi}{2},\ \frac{5\pi}{6},\ \frac{3\pi}{2}\right\}$

Have this problem put upon Board by Student. Stress use of $\cos^{-1}$ to put in proper form for Cos Add formula

21. $\sin^2\left(x+\frac{\pi}{6}\right)-\cos^2\left(x+\frac{\pi}{6}\right)=\frac{\sqrt{2}}{2}$

using Def of Cos ADD.

$x_1=x+\frac{\pi}{6}$ $x_2=x+\frac{\pi}{6}$

$\therefore\ \cos x_1+x_2=\cos 2\left(x+\frac{\pi}{6}\right)$

$-\cos 2\left(x+\frac{\pi}{6}\right)=\frac{\sqrt{2}}{2}$

$\cos\left(2x+\frac{\pi}{3}\right)=-\frac{\sqrt{2}}{2}$

$0\le x\le 2\pi$

$0\le 2x\le 4\pi$

$\frac{\pi}{3}\le 2x+\frac{\pi}{3}\le 4\pi+\frac{\pi}{3}$

$2x+\frac{\pi}{3}=\frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{11\pi}{4},\ \frac{13\pi}{4}$

$2x+\frac{\pi}{3}=\frac{3\pi}{4}$

$2x=\frac{3\pi}{4}-\frac{\pi}{3}$

$=\frac{9\pi}{12}-\frac{4\pi}{12}$

$=\frac{5\pi}{12}$

$x=\frac{5\pi}{24}$

$2x+\frac{\pi}{3}=\frac{5\pi}{4}$

$2x=\frac{5\pi}{4}-\frac{\pi}{3}$

$=\frac{15\pi-4}{12}$

$x=\frac{11\pi}{24}$

$x\in\left\{\frac{5\pi}{24},\ \frac{11\pi}{24},\ \frac{29\pi}{24},\ \frac{35\pi}{24}\right\}$

#1

$2\sec x - 2\sqrt{2} = 0$

$2\sec x = 2\sqrt{2}$

$\sec x = \sqrt{2}$

$\cos x = \frac{\sqrt{2}}{2}$

$x \in \left\{\frac{\pi}{4}, \frac{7\pi}{4}\right\}$

#2 $2 + \sec x = 0$

$\sec x = -2$

$\cos x = -\frac{1}{2}$

$x \in \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

#3 $\tan^2 x = 1$

$\tan x = \pm 1$

$x \in \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

#4 $\cot^2 x = 1$

$\cot x = \pm 1$

↩

#5 $\tan^2 x + \tan x = 0$

$\tan x(\tan x + 1) = 0$

$\tan x = 0 \quad \tan x = -1$

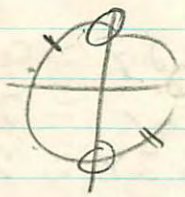
$\left\{0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}\right\}$

#6 $2\cot^2 x + 2\cot x = 0$

$2\cot x(\cot x + 1) = 0$

$\cot x = 0 \quad \cot x = -1$

$\left\{\frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\right\}$



#7 $2\sin x \sec x = \sec x$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

$\sec x = 0$

but $\sec x \neq 0$ for any value of x in Domain.

#8 $3\sec x - \cos x = 2$

$3 - \cos^2 x = 2\cos x$

$\cos^2 x + 2\cos x - 3 = 0$

$(\cos x + 3)(\cos x - 1) = 0$

$\cos x = -3$ or $\cos x = +1$

$\{0\}$

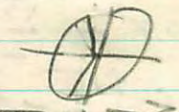
but $\cos x \neq -3$ $\therefore \cos x = 1 \Rightarrow$

#9 $2\sec^2 x + 3\sec x - 2 = 0$

$(2\sec x - 1)(\sec x + 2) = 0$

$\sec x = \frac{+1}{2}$ $\sec x = -2$

$\cos x = \frac{+1}{2}$



$\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

10) $\tan^2 x + 2\tan x + 1 = 0$

$(\tan x + 1)^2 = 0$

$\tan x = -1$

$\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

11) $3\tan\frac{x}{2} = \cot\frac{x}{2}$

$3\tan^2\frac{x}{2} = 1$ $\quad 0 \le x < 2\pi$

$\tan^2\frac{x}{2} = \frac{1}{3}$ $\quad 0 \le \frac{x}{2} < \pi$

$\tan\frac{x}{2} = \pm\frac{\sqrt{3}}{3}$

$\frac{x}{2} = \frac{\pi}{6}$ $\quad \frac{x}{2} = \frac{5\pi}{6}$

$x = \frac{\pi}{3}$ $\quad x = \frac{5\pi}{3}$

Both are good!

12) $\frac{1}{2}\tan 2x = \cos x$

$\tan 2x = 2\cos x$

$\sin 2x = 2\cos x \cos 2x$

$2\sin x \cos x = 2\cos x \cos 2x$ $\quad \cos x = 0$

$\sin x = \cos 2x$ $\quad$ no its OK

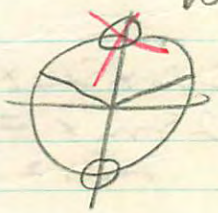
$\sin x = \cos^2 x - \sin^2 x$

$-\sin x = 1 - \sin^2 x - \sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ $\quad \sin x = -1$



~~$\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$~~

OK

$\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

Demo

13) $\sec^2 2x = 1 - \tan 2x$

$1 + \tan^2 2x = 1 - \tan 2x$

$\tan^2 2x + \tan 2x = 0$

$\tan 2x (1 + \tan 2x) = 0$

$\tan 2x = 0 \quad \tan 2x = -1$

$2x \in \left\{0, \pi, 2\pi, 3\pi, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}\right\}$

$\therefore$

$x \in \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}\right\}$

14) $\tan^2 x = \sin 2x$

$\tan^2 x = 2 \sin x \cos x$

$\frac{\sin^2 x}{\cos^2 x} = 2 \sin x \cos x$

$\sin x = 2 \cos^2 x \cos x$

$\sin x = 2 \cos^3 x$

$\sin^2 x = 4 \cos^6 x$

$1 - \cos^2 x = 4 \cos^6 x$

$4 \cos^6 x + \cos^2 x - 1 = 0$

$(2 \cos^3 x - 1)($

redone

$\sin x = 0$

$1 - \cos^2 x = 2 \sin x \cos^3 x$

$1 = 2 \sin x \cos^3 x + \cos^2 x$

$1 = (2 \sin x \cos x (+1) \cos^2 x$

multiply by $\sin 2x$ $\Rightarrow$ yields 3 line sol.

15) $2\cot x = 3\csc 2x$

$2\sin 2x = 3\tan x$

$4\sin x\cos x = \dfrac{3\sin x}{\cos x}$ $\quad \sin x = 0$ but $\sin x \neq 0$

$4\cos^2 x = 3$

$\cos^2 x = \dfrac{3}{4}$

$\cos x = \pm\dfrac{\sqrt{3}}{2}$

$\left\{\cancel{0}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \cancel{\pi}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right\}$

16) $\sqrt{3}\tan x + 1 = \sec x$

$\sqrt{3}\dfrac{\sin x}{\cos x} + 1 = \dfrac{1}{\cos x}$

$\sqrt{3}\sin x + \cos x = 1$

$\sqrt{3}\sin x = 1 - \cos x$

$3\sin^2 x = 1 - 2\cos x + \cos^2 x$

$3(1-\cos^2 x)$

$3 - 3\cos^2 x =$

$4\cos^2 x - 2\cos x - 2 = 0$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\dfrac{1}{2}$ $\quad \cos x = 1$

$\left\{0, \dfrac{2\pi}{3}, \cancel{\dfrac{4\pi}{3}}\right\} \Rightarrow \left\{0, \dfrac{2\pi}{3}\right\}$

$\sin\dfrac{4\pi}{3} = -\dfrac{\sqrt{3}}{2}$

$\cos\dfrac{4\pi}{3} = -\dfrac{1}{2}$

$\dfrac{\sqrt{3}}{2}, -2$

check

$\sqrt{3}\tan 0 + 1 = \sec 0$

$1 = 1$

$\sqrt{3}\tan\dfrac{2\pi}{3} + 1 = \sec\dfrac{2\pi}{3}$

$-\sqrt{3}\cdot(-\sqrt{3}) + 1 =$

$-3 + 1 =$

$-2 = -2$

$\sqrt{3}\sqrt{3} + 1 = -2$

$4 = -2$

Pg 156-157

(14) $\tan^2 x = \sin 2x$

$\frac{\sin^2 x}{\cos^2 x} = 2\sin x \cos x$

$\frac{\sin x}{\cos^2 x} = 2\cos x$ or $\sin x = 0$

$\sin^2 x = 4\cos^6 x$

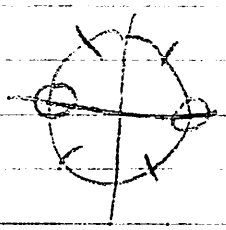
$4\cos^6 x + \cos^2 x - 1 = 0$

$(2\cos^2 x - 1)(2\cos^4 x + \cos^2 x + 1) = 0$

$\downarrow\downarrow$ Never 0

$\cos^2 x = \frac{1}{2}$ $0, \pi$

$\cos x = \pm\frac{\sqrt{2}}{2} \Rightarrow \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$



$\tan^2 \frac{\pi}{4}$

$$\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$$

$$\cos\left(\tan^{-1}(1+x)\right)\cos\left(\tan^{-1}(1-x)\right)$$

$$- \sin\left(\tan^{-1}(1+x)\right)\sin\left(\tan^{-1}(1-x)\right) = 0$$

$$\frac{1}{\sqrt{1+(1+x)^2}} \cdot \frac{1}{\sqrt{1+(1+x)^2}} - \frac{1+x}{\sqrt{1+(1+x)^2}} \cdot \frac{1-x}{\sqrt{1+(1-x)^2}} = 0$$

$$\frac{1-(1-x^2)}{\sqrt{1+(1+x)^2}\ \sqrt{1+(1-x)^2}} = 0$$

$$\frac{x^2}{\sqrt{\ }\sqrt{\ }} = 0$$

$$x = 0$$

17 $\{\frac{\pi}{3}, \frac{5\pi}{3}, \pi\}$

$\cos 2x \sec x + 1 = 0$

$\cos 2\pi \sec \pi + 1 = 0$

$1(-1) + 1 = 0$

$-1 + 1 = 0$

$0 = 0$

$\cos^{2\pi}_{3} \sec \frac{\pi}{3} + 1 = 0$

$-\frac{1}{2} \cdot 2 + 1 = 0$

$-1 + 1 = 0$

$0 = 0$

$\cos \frac{4\pi}{3} \sec \frac{5\pi}{3} + 1 = 0$

$-\frac{1}{2} \cdot 2 + 1 = 0$

$-1 + 1 = 0$

$0 = 0$

18) $\tan^{-1}(x-2) = \frac{\pi}{3}$

$\tan \frac{\pi}{3} = x-2$

$\frac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}} = x-2$

$\frac{\sqrt{3}}{2} \cdot 2 = x-2$

$\sqrt{3} = x-2$

$x = 2+\sqrt{3}$

19) $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$ Cannot operate w/ tan since $\tan \frac{\pi}{2}$ undefined

3 solutions also $\pm\sqrt{2}$

See pg 148 # 32, 33

$\cot^{-1} x = \tan^{-1}(\frac{1}{x})$

$\tan^{-1} x = \frac{\pi}{2} - \cot^{-1} x = \frac{\pi}{2} - \tan^{-1}(\frac{1}{x})$

$\tan^{-1} x + \tan^{-1}(\frac{1}{x}) = \frac{\pi}{2}$

let $y = x$ $\tan^{-1} x + \tan^{-1}(\frac{1}{y}) = \frac{\pi}{2}$ compare w/ this

$1+x = y \Rightarrow 1-x = \frac{1}{y}$

$\frac{1}{y} = \frac{1}{x+1}$ $1-x = \frac{1}{x+1} \Rightarrow 1-x^2 = 1 \therefore x^2 = 0, x = 0$

157

20 $\cot x \cos 2x = 3$

$\cos 2x = 3\tan x$

$\frac{2\sin x \cos x}{\cos^2 x - \sin^2 x} = \frac{3\sin x}{\cos x}$

$\frac{2\sin x \cos^2 x}{\cos^2 x - \sin^2 x} = 3\sin x$

$2\sin x \cos^2 x = 3\sin x(\cos^2 x - \sin^2 x)$ $\sin x = 0$, $\sin x \neq 0$

$2\cos^2 x = 3\cos^2 x - 3\sin^2 x$

$3\sin^2 x = \cos^2 x$

$3 - 3\cos^2 x = \cos^2 x$

$4\cos^2 x = 3$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

~~0, π~~

$\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

21 $\cos 2x \tan x = 1$

$\frac{\sin 2x}{\cos 2x} = \frac{\cos x}{\sin x}$

$\frac{2\sin^2 x \cos x}{\cos^2 x - \sin^2 x} = \cos x$ $\cos x \neq 0$

$2\sin^2 x = \cos^2 x - \sin^2 x$

$3\sin^2 x = 1 - \sin^2 x$

$4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

$\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

22

$\sin 2x \tan^2 2x - \tan 2x = \sin 2x \quad \sin 2x = 0$

$\tan^2 2x - \frac{1}{\cos 2x} = 1$

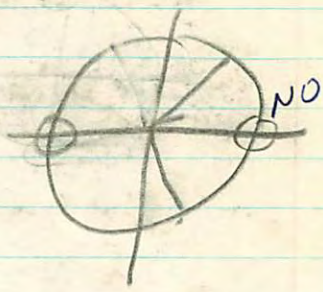
$\sin^2 2x - \cos 2x = \cos^2 2x$

$1 - \cos^2 2x - \cos 2x = \cos^2 2x$

$2\cos^2 2x + \cos 2x - 1 = 0$

$(2\cos 2x - 1)(\cos 2x + 1) = 0$

$\cos 2x = \frac{1}{2} \quad \cos 2x = -1$



$2x \in \{\cancel{0}, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, \cancel{2\pi}, \frac{7\pi}{3}, 3\pi, \frac{11\pi}{3}\}$ $0 \le 2x \le 4\pi$ $\cos 2x \neq 0$

$x \in \{\cancel{0}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \cancel{\pi}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\}$.

23) $\tan\left(\frac{\pi}{4} + 3x\right) = 2\tan 6x$

$\frac{\tan\frac{\pi}{4} + \tan 3x}{1 - \tan\frac{\pi}{4}\tan 3x} = 2\tan(3x+3x)$

$\frac{1}{2} \cdot \frac{1 + \tan 3x}{1 - \tan 3x} = \frac{4\tan 3x}{1 - \tan^2 3x}$

$(1 + \tan 3x)^2 = 4\tan 3x$

$\tan^2 3x - 2\tan 3x + 1 = 0$

$(\tan 3x - 1)^2 = 0$

$\tan 3x = 1 \quad 0 \le 3x < 6\pi$

Impossible since this causes zero in denominator

Typical value of x would be $\frac{\pi}{12}$. This leads to $2\tan\left(6 \cdot \frac{\pi}{12}\right) = 2\tan\frac{\pi}{2}$ which is not defined.

Chap T Test

① $\sin Y = \frac{\sqrt{3}}{2}$

$Y = \left\{\frac{\pi}{3} + \frac{2k\pi}{3}, \frac{2\pi}{3} + 2k\pi\right\}$

② $\sin(\cos^{-1}\frac{1}{2})$

$\cos^{-1}\frac{1}{2} = Y$

$\cos Y = \frac{1}{2}$

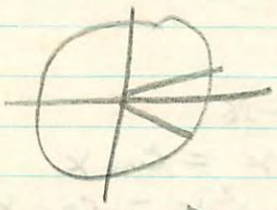
$\sin^2 Y + \frac{1}{4} = 1$

$\sin^2 Y = \frac{3}{4}$

$\sin Y = \frac{\sqrt{3}}{2}$

③ $\sec y = \frac{2}{\sqrt{3}}$

$\cos y = \frac{\sqrt{3}}{2}$



$y \in \left\{\frac{\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi\right\}$

④ $\tan^{-1}(\sin \pi)$

$\tan^{-1} 0 = X$

$\tan X = 0$

$X = 0$ range $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

#5

$$\sin\left(2\cos^{-1}\frac{5}{12}\right)$$

$$2\sin\left(\cos^{-1}\frac{5}{12}\right)\cos\left(\cos^{-1}\frac{5}{12}\right)$$

$$2\left[\frac{\frac{5}{12}}{\sqrt{1+\frac{25}{144}}}\right]\left[\frac{1}{\sqrt{1+\frac{25}{144}}}\right]$$

$$2\left[\frac{\frac{5}{12}}{\frac{13}{12}}\right]\left[\frac{1}{\frac{13}{12}}\right] = 2\left[\frac{5}{13}\right]\left[\frac{12}{13}\right] = \frac{120}{169}$$

#6

$$\cos^2 x = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2} + 2k\pi$$

#7

$$\cos 2x = \cos x$$

$$\cos^2 x - \sin^2 x = \cos x$$

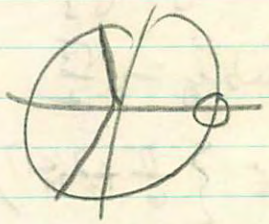
$$\cos^2 x - 1 + \cos^2 x = \cos x$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1)$$

$$\cos x = -\frac{1}{2} \quad \cos x = 1$$

$$x \in \left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$$



#8 $3\cos^2 x = 1 - \frac{5}{2}\sin x$

$3 - 3\sin^2 x = 1 - \frac{5}{2}\sin x$

$3\sin^2 x - \frac{5}{2}\sin x - 2 = 0$

$6\sin^2 x - 5\sin x - 4 = 0$

$(\sin x = \frac{4}{3} \quad \sin x = -\frac{1}{2}$

$\sin x \not> 1$!!

$\sin x = -\frac{1}{2}$

$x \in \left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

$6u^2 - 5u - 4 = 0$

$\frac{5 \pm \sqrt{25+96}}{12}$

$\frac{5 \pm 11}{12}$

$\frac{16}{12}, -\frac{6}{12}$

$\frac{4}{3}, -\frac{1}{2}$

#9 $\tan x = \frac{1}{2}\sec x$

$\frac{\sin x}{\cos x} = \frac{1}{2\cos x}$ $\quad \cos x \neq 0$

$\sin x = \frac{1}{2}$

$\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

#10 $\sqrt{3}\csc^2 x + 2\csc x = 0$

$\csc x(\sqrt{3}\csc x + 2) = 0$

$\csc x = 0 \quad \csc x = -\frac{2}{\sqrt{3}}$

$\csc x \neq 0 \quad \sin x = -\frac{\sqrt{3}}{2}$

$\left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

22 $\cos^2(x+\frac{\pi}{3}) - \sin^2(x+\frac{\pi}{3}) = \frac{1}{2}$

$\cos 2(x+\frac{\pi}{3}) = \frac{1}{2}$

$2x+\frac{2\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}$

$0 \le x < 2\pi$

$\frac{2\pi}{3} \le 2x+\frac{2\pi}{3} < 4\pi+\frac{2\pi}{3}$

$2(x+\frac{\pi}{3}) \in \{\frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}\}$

$2x+\frac{2\pi}{3} = \frac{5\pi}{3}$

$2x = \pi$

$x = \frac{\pi}{2}$

$2x+\frac{2\pi}{3} = \frac{7\pi}{3}$

$2x = \frac{5\pi}{3}$

$x = \frac{5\pi}{6}$

$2x+\frac{2\pi}{3} = \frac{11\pi}{3}$

$2x = 3\pi$

$x = \frac{3\pi}{2}$

$2x+\frac{2\pi}{3} = \frac{13\pi}{3}$

$2x = \frac{11\pi}{3}$

$x = \frac{11\pi}{6}$

$x \in \{\frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\}$

23

$\sin 2x - \sin 4x = 2\sin x$

$\sin 2x - 2\sin 2x\cos 2x = 2\sin x$

$2\sin x\cos x - 4\sin x\cos x\cos 2x = 2\sin x$

$\cos x - 2\cos x\cos 2x = 1$

$\cos x - 2\cos x(\cos^2 x - \sin^2 x) = 1$

$\cos x - 2\cos x(\cos^2 x - 1 + \cos^2 x) = 1$

$\cos x - 2\cos x(2\cos^2 x - 1) = 1$

$\cos x - 4\cos^3 x + 2\cos x = 1$

$-4\cos^3 x + 3\cos x - 1 = 0$

$(\cos x+1)(-4\cos^2 x + 4\cos x - 1) = 0$

$-1(\cos x+1)(4\cos^2 x - 4\cos x + 1)$

$-1(\cos x+1)(2\cos x - 1)^2 = 0$

$\cos x + 1 \qquad \cos x = \frac{1}{2}$

$\sin x = 0$

$-4u^2+4u-1$

$u+1 \;|\; -4u^3+3u-1 = 0$

$-4u^3-4u^2$

$+4u^2+3u$

$+4u^2+4u$

$-u-1$

$-u-1$

$x = K$

$x = \frac{\pi}{3} + 2K\pi$

$x = \frac{5\pi}{3} + 2K\pi$

(24)

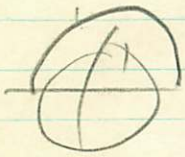
$x = 5$

$x^2 = 25$

$x^2 - 25 = 0$

$(x-5)(x+5) = 0$

$x \in \{\pm 5\}$



(25) $\cos^{-1} 2x = \frac{\pi}{3} - \cos^{-1} x$

$2x = \cos\frac{\pi}{3}\cos(\cos^{-1}x) + \sin\frac{\pi}{3}\sin(\cos^{-1}x)$

$2x = \frac{1}{2}x + \frac{\sqrt{3}}{2}\sqrt{1-x^2}$

$4x = x + \sqrt{3}\sqrt{1-x^2}$

$3x = \sqrt{3}\sqrt{1-x^2}$

$9x^2 = 3(1-x^2)$

$12x^2 = 3$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$ reject negative

$\cos^{-1}(1) = \frac{\pi}{3} - \cos^{-1}(\frac{1}{2})$

$0 = \frac{\pi}{3} - \frac{\pi}{3}$

$\cos^{-1}(-1) = \frac{\pi}{3} - (\cos^{-1}(-\frac{1}{2}))$

$\pi = \frac{\pi}{3} - (-\frac{2\pi}{3})$

$\pi = \pi$

FREEDOM COVER SHEET

Name C. Heston

Per _____

1) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$	2) $\frac{\pi}{2}$	3) $\frac{\pi}{2}, \frac{3\pi}{2}$	4) $0, \frac{\pi}{2}, \pi$
5) $\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}$	6) $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$	7) $0, \frac{\pi}{2}, \pi$	8) ϕ
9) $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$	10) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$	11) $0\ \pi\ \frac{7\pi}{6}\ \frac{11\pi}{6}$	12) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
13) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	14) $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$	15) $\frac{2\pi}{3}, \frac{4\pi}{3}$	16) $\frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$
17) π	18) $0, \pi$	19) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$	20) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
21) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$	22) $\frac{7\pi}{6}, \frac{11\pi}{6}$	23) $0, \frac{3\pi}{2}$	24) $\frac{3\pi}{2}$
25) $\frac{\pi}{6}, \frac{7\pi}{6}$	26) ϕ	27) $\frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$	28) $0\ \frac{\pi}{2}\ \pi$
29) $0, \pi$	30) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$	31) $\frac{\pi}{2}, \frac{3\pi}{2}$	32) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$
33) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$	34) $0, \pi$	35) $\frac{\pi}{2}, \frac{3\pi}{2}$	36) $\lvert x\rvert \le 1$
37) $-\frac{\pi}{2} < x < \frac{\pi}{2}$	38) $\lvert x\rvert \ge 1$	39) $\lvert x\rvert \ge 1$	40) $\frac{\pi}{2}$
41) $\frac{\pi}{6}, \frac{5\pi}{6}$	42) $\frac{\pi}{3}, \frac{5\pi}{3}$	43) $\frac{5\pi}{24}, \frac{11\pi}{24}, \frac{29\pi}{24}, \frac{35\pi}{24}$	44) $0, \frac{2\pi}{3}, \frac{4\pi}{3}$
45) $\frac{7\pi}{6}, \frac{11\pi}{6}$	46) $0, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{7\pi}{8}, \pi, \frac{11\pi}{8}, \frac{3\pi}{2}, \frac{15\pi}{8}$	47) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$	48) $\frac{\pi}{6}, \frac{5\pi}{6}$
49) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	50) $\lvert x\rvert \le \frac{\pi}{2}$	51) $0 \le x \le \pi$	52) $x \in \mathbb{R}$
53) $\lvert x\rvert \le \frac{\pi}{2}, x \ne 0$	54) $0 < x < \pi$	55) $\frac{\pi}{4}, \frac{7\pi}{4}$	56) $\frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$
57) $0\ \frac{\pi}{2}$	58) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$	59) $\frac{\pi}{2}, \frac{3\pi}{2}$	60) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$
61) $x \in \mathbb{R}$	62) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$	63) $\lvert x\rvert \le 1$	64) $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{\pi}{2}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}$
65) 0	66) $0 \le x \le \pi\ x \ne \frac{\pi}{2}$	67) $\frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{4}$	68) $\frac{u}{\sqrt{1+u^2}}$
69) $x \in \mathbb{R}$	70) 0	71) $\frac{1}{\sqrt{1+u^2}}$	72) $\frac{2\pi}{3}, \frac{4\pi}{3}$
73) -2	74) $\frac{\sqrt{1-u^2}}{u}$	75) $\sqrt{1-u^2}$	76) $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{\pi}{2}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}$
77)	78)	79)	80)
81)	82)	83)	84)
85)	86)	87)	88)
89)	90)	91)	92)
93)			

FREEDOM COVER SHEET

Name _____

Per _____

1) _____	2) _____	3) _____	4) _____
5) _____	6) _____	7) _____	8) _____
9) _____	10) _____	11) _____	12) _____
13) _____	14) _____	15) _____	16) _____
17) _____	18) _____	19) _____	20) _____
21) _____	22) _____	23) _____	24) _____
25) _____	26) _____	27) _____	28) _____
29) _____	30) _____	31) _____	32) _____
33) _____	34) _____	35) _____	36) _____
37) _____	38) _____	39) _____	40) _____
41) _____	42) _____	43) _____	44) _____
45) _____	46) _____	47) _____	48) _____
49) _____	50) _____	51) _____	52) _____
53) _____	54) _____	55) _____	56) _____
57) _____	58) _____	59) _____	60) _____
61) _____	62) _____	63) _____	64) _____
65) _____	66) _____	67) _____	68) _____
69) _____	70) _____	71) _____	72) _____
73) _____	74) _____	75) _____	76) _____
77) _____	78) _____	79) _____	80) _____
81) _____	82) _____	83) _____	84) _____
85) _____	86) _____	87) _____	88) _____
89) _____	90) _____	91) _____	92) _____
93) _____			

FREEDOM COVER SHEET

Name _____

Per _____

1) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

2) $\frac{\pi}{2}$

3) $\frac{\pi}{2}\ \frac{3\pi}{2}$

4) $0\ \pi\ 0, \frac{\pi}{2}\ \pi$

5) $\frac{\pi}{4}\ \frac{\pi}{2}\ \frac{5\pi}{4}\ \frac{3\pi}{2}$

6) $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

7) $0, \frac{\pi}{2}, \pi$

8) ϕ

9) $0\ \frac{\pi}{3}\ \frac{2\pi}{3}\ \pi$

10) $\frac{\pi}{4}\ \frac{5\pi}{4}$ AND $\frac{5\pi}{4}\ \frac{7\pi}{4}$

11) $0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

12) $\frac{\pi}{4}\ \frac{3\pi}{4}\ \frac{5\pi}{4}\ \frac{7\pi}{4}$

13) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

14) $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

15) $\frac{2\pi}{3}\ \frac{4\pi}{3}$

16) $\frac{\pi}{2}\ \frac{2\pi}{3}\ \frac{4\pi}{3}\ \frac{3\pi}{2}\ \frac{\pi}{3}\ \frac{\pi}{2}\ \frac{3\pi}{2}\ \frac{5\pi}{3}$

17) π

18) $0\ \pi$

19) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

20) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

21) $\frac{\pi}{3}\ \frac{2\pi}{3}\ \frac{4\pi}{3}\ \frac{5\pi}{3}$

22) $\frac{7\pi}{6}\ \frac{11\pi}{6}$

23) $0\ \frac{3\pi}{2}$

24) $\frac{3\pi}{2}$

25) $\frac{5\pi}{6}\ \frac{11\pi}{6}\ \frac{\pi}{6}, \frac{7\pi}{6}$

26) ϕ

27) $\frac{\pi}{3}\ \frac{\pi}{2}\ \frac{3\pi}{2}\ \frac{5\pi}{3}$

28) $0, \pi$ and $\frac{\pi}{2}$

29) $0\ \pi$

30) $\frac{\pi}{2}\ \frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

31) $\frac{\pi}{2}, \frac{3\pi}{2}$

32) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

33) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

34) $0\ \pi$

35) $\frac{\pi}{2}\ \frac{3\pi}{2}$

36) $|x| \le 1$

37) $x \in \mathbb{R}\ \ -\frac{\pi}{2} < x < \frac{\pi}{2}$

38) $|x| \ge 1$

39) $|x| \ge 1$

40) $\phi\ \ \frac{\pi}{2}$

41) $\frac{\pi}{6}, \frac{5\pi}{6}$

42) $\frac{\pi}{3}, \frac{5\pi}{3}$

43) $\frac{5\pi}{24}\ \frac{11\pi}{24}\ \frac{29\pi}{24}\ \frac{35\pi}{24}$

44) $0\ \frac{2\pi}{3}\ \frac{4\pi}{3}$

45) $\frac{7\pi}{6}\ \frac{11\pi}{6}$

46) $0\ \frac{3\pi}{8}\ \frac{\pi}{2}\ \frac{7\pi}{8}\ \frac{11\pi}{8}\ \frac{3\pi}{2}\ \frac{15\pi}{8}$

47) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

48) $\frac{\pi}{6}\ \frac{5\pi}{6}$

49) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

50) $|x| \le \frac{\pi}{2}$

51) $0 \le x \le \pi$

52) $x \in \mathbb{R}$

53) $|x| \le \frac{\pi}{2}\ \ x \neq 0$

54) $0 < x < \pi$

55) $\frac{\pi}{4}, \frac{7\pi}{4}$

56) $\frac{\pi}{2}\ \frac{3\pi}{4}\ \frac{3\pi}{2}\ \frac{7\pi}{4}$

57) $0\ \frac{\pi}{2}$

58) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

59) $\frac{\pi}{2}\ \frac{3\pi}{2}$

60) $\frac{\pi}{2}\ \frac{\pi}{6}\ \frac{11\pi}{6}\ \frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

61) $x \in \mathbb{R}$

62) $\frac{7\pi}{6}, \frac{11\pi}{6}$, and $\frac{\pi}{2}$

63) $|x| \le 1$

64) $0\ \frac{\pi}{4}\ \frac{\pi}{2}\ \pi\ \frac{5\pi}{4}\ \frac{3\pi}{2}$

65) 0

66) $0 \le x \le \pi\ \ x \neq \frac{\pi}{2}$

67) $\frac{\pi}{6}\ \frac{3\pi}{4}\ \frac{5\pi}{4}\ \frac{7\pi}{4}$

68) $\frac{u}{\sqrt{1+u^2}}$

69) $\mathbb{R}$

70) 0

71) $\frac{1}{\sqrt{1+u^2}}$

72) $\frac{2\pi}{3}\ \frac{4\pi}{3}$

73) $2\ \ -2$

74) $\frac{\sqrt{1-u^2}}{4}$

75) $\sqrt{1-u^2}$

76) $0\ \frac{\pi}{4}\ \frac{\pi}{2}\ \pi\ \frac{5\pi}{4}\ \frac{3\pi}{2}$

77) $\pm\frac{3\sqrt{10}}{10}$

78) $\sqrt{1-u^2}$

79) $\frac{5\sqrt{3}+12}{26}$

80) $\frac{2\sqrt{5}}{5}$

81) $\pm\sqrt{\frac{34-5\sqrt{34}}{68}}$

82) Identity $x \neq k\pi$

83) $\frac{\pi}{2}, \frac{3\pi}{2}$

84) $\frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{3\pi}{2}\ \frac{11\pi}{6}$

85) $0\ \frac{\pi}{4}\ \frac{\pi}{2}\ \pi\ \frac{5\pi}{4}\ \frac{3\pi}{2}$

86) 0

87) $\frac{\sqrt{1-u^2}}{u}$

88) $\frac{5}{4}, 4$

89) $x \in \mathbb{R}\ \ x \neq \left\{\frac{(2k+1)\pi}{2}\right\}$

90) $\frac{1}{\sqrt{u^2-1}}$

91) 16

92) $x \in \mathbb{R}\ \ x \neq \left\{\frac{(2k+1)\pi}{2}\right\}$

93) $\frac{\sqrt{u^2-1}}{u}$

84 → $\frac{5\pi}{3}\ \frac{11\pi}{6}$

37) $\operatorname{Tan}(\operatorname{Tan}^{-1} x) = x$

38) $\operatorname{Sec}(\operatorname{Sec}^{-1} x) = x$

39) $\operatorname{Csc}(\operatorname{Csc}^{-1} x) = x$

40) $\operatorname{Tan}^{-1} x + \operatorname{Tan}^{-1} 1 = \frac{3\pi}{4}$

41) $2 \operatorname{Sin} x \operatorname{Sec} x = \operatorname{Sec} x$

42) $3 \operatorname{Tan}\frac{x}{2} = \operatorname{Cot}\frac{x}{2}$

43) $\operatorname{Sin}^2(x+\frac{\pi}{6}) - \operatorname{Cos}^2(x+\frac{\pi}{6}) = \frac{\sqrt{2}}{2}$

44) $\operatorname{Cos} 2x + 2 \operatorname{Sin}^2\frac{x}{2} = 1$

45) $\operatorname{Csc}^2 x + 2 \operatorname{Csc} x = 0$

46) $\operatorname{Sec}^2 2x + \operatorname{Tan} 2x = 1$

47) $2 \operatorname{Sin}^2 x - 3 \operatorname{Sin} x + 1 = 0$

48) $2 \operatorname{Sin}^2 x + 3 \operatorname{Sin} x - 2 = 0$

49) $\operatorname{Cot} x \operatorname{Tan} 2x = 3$

50) $\operatorname{Sin}^{-1}(\operatorname{Sin} x) = x$

51) $\operatorname{Cos}^{-1}(\operatorname{Cos} x) = x$

52) $\operatorname{Tan}(\operatorname{Tan}^{-1} x) = x$

53) $\operatorname{Csc}^{-1}(\operatorname{Csc} x) = x$

54) $\operatorname{Cot}^{-1}(\operatorname{Cot} x) = x$

55) $2 \operatorname{Sec} x - 2\sqrt{2} = 0$

56) $2 \operatorname{Cot}^2 x + 2 \operatorname{Cot} x = 0$

57) $\operatorname{Sin} x + \operatorname{Cos} x = 1$

58) $\operatorname{Sin} 2x \operatorname{Sin} x - \operatorname{Cos} 2x \operatorname{Cos} x = 1$

59) $2 \operatorname{Sin}(x - \frac{\pi}{6}) = \sqrt{3} \operatorname{Sin} x$

60) $2 \operatorname{Sin}^2 x - \operatorname{Sin} x - 1 = 0$

61) $\operatorname{Sin}^2(x+\frac{\pi}{2}) + \operatorname{Cos}^2(x+\frac{\pi}{2}) - 1 = 0$

62) $2 \operatorname{Cos} 2x + \operatorname{Sin} 2x \operatorname{Sec} x = 0$

63) $\operatorname{Cos}(\operatorname{Cos}^{-1} x) = x$

64) $2 \operatorname{Cos}^2 x \operatorname{Sin}^2 x - \operatorname{Cos} x \operatorname{Sin} x = 0$

65) $3 \operatorname{Sec} x - \operatorname{Cos} x = 2$

66) $\operatorname{Sec}^{-1}(\operatorname{Sec} x) = x$

) $2 \operatorname{Tan} x \operatorname{Sin} x + 2 \operatorname{Sin} x = \operatorname{Tan} x + 1$

68) $\operatorname{Sin}(\operatorname{Tan}^{-1} U) = x$

69) $\operatorname{Cot}(\operatorname{Cot}^{-1} x) = x$

70) $\operatorname{Sin} 2x \operatorname{Sin} x + \operatorname{Cos} 2x \operatorname{Cos} x = 1$

71) $\operatorname{Cos}(\operatorname{Tan}^{-1} U) = x$

72) $2 + \operatorname{Sec} x = 0$

73) $\operatorname{Tan}(\operatorname{Tan}^{-1} x + \operatorname{Tan}^{-1} 1) = -3$

74) $\operatorname{Tan}(\operatorname{Cos}^{-1} U) = x$

75) $\operatorname{Cos}(\operatorname{Sin}^{-1} U) = x$

76) $2 \operatorname{Cos}^2 x \operatorname{Sin}^2 x - \operatorname{Cos} x \operatorname{Sin} x = 0$

77) $\operatorname{Cos}(\frac{1}{2} \operatorname{ArcTan}\frac{3}{4}) = x$

78) $\operatorname{Sin}(\operatorname{Cos}^{-1} U) = x$

79) $\operatorname{Cos}[\operatorname{ArcSin}(-\frac{1}{2}) + \operatorname{Cos}^{-1}\frac{5}{13}] = x$

80) $\operatorname{Tan}(\pi + \operatorname{ArcSin}\frac{2}{3}) = x$

81) $\operatorname{Sin}(\frac{1}{2} \operatorname{ArcTan}\frac{3}{5}) = x$

82) $\operatorname{Cot}^2 x - \operatorname{Cos}^2 x = \operatorname{Cot}^2 x \operatorname{Cos}^2 x$

83) $2 \operatorname{Cos} x = 4 \operatorname{Cos} x \operatorname{Csc} x$

84) $\operatorname{Tan} 2x = 3 \operatorname{Cot} 2x$

85) $\operatorname{Cos}^3 x - \operatorname{Sin}^3 x = (\operatorname{Cos} x - \operatorname{Sin} x)(1 + \frac{1}{2} \operatorname{Sin} 2x)$

86) $\operatorname{Cot}(\operatorname{Cos}^{-1} x) = x$

87) $\operatorname{Sin}(\operatorname{Sec}^{-1} x) = x$

88) $\operatorname{Sec}^{-1}\frac{5}{3} + \operatorname{Csc}^{-1}\frac{5}{4} = 2 \operatorname{Tan}^{-1}\frac{x}{3}$

89) $\operatorname{Tan}^2 x - \operatorname{Sec}^2 x = -1$

90) $\operatorname{Tan}(\operatorname{Csc}^{-1} U) = x$

) $\operatorname{Sin}^{-1}\frac{3}{5} + \operatorname{Sin}^{-1}\frac{12}{13} = \operatorname{Cos}^{-1}\frac{-x}{65}$

92) $1 - 2 \operatorname{Sin}^2 x = \frac{1 - \operatorname{Tan}^2 x}{1 + \operatorname{Tan}^2 x}$

93) $\operatorname{Cos}(\operatorname{Csc}^{-1} U) = x$

HONORS TRIGONOMETRY

1776

CHAPTER 4

"FREEDOM RINGS"

There are 76 required problems. Total value is 10 assignments of " + ". Each error or omission reduces the 10 by 1 level each. (total 10 wrong = 10 checks; total 20 wrong = 10 zeros; etc.) An error in recording your answer on the answer sheet IS an error. An answer on the answer sheet without the proper work is an error on the answer sheet AND an error on the work ! (i.e. 2 errors) There are 17 "extra" problems. These will forgive up to 6 errors.

Some problems will have obvious methods; others will (multiple angle problems for example) need explanation justifying method. When in doubt, explain. The work MUST be neat, in order, and easy to read. Comments and/ or explanations are to be adjacent to their problem. Each page is to have your name and period number in the upper right hand corner. After a problem is solved, write the answer ONLY (not in set notation) on the cover sheet. The complete answer, in proper notation is to be at the bottom of its problem.

Solve for $x \in$ of the Reals or for all $x \in [0, 2\pi)$ where appropriate. In the case of "inverse" trig functions, your solution is to be with the proper range and/ or domain of that particular function. Include "circle solutions" in addition to proper set notation where appropriate.

Fill in the cover page with your name, period and numbered 1 through 93 with the answers ONLY to each problem OR leave a blank if you could not do the problem. Attach this cover sheet on the top of all your work. The entire packet will be turned in the day you take the chapter 4 exam. (It is due when you enter the room for the exam NOT after you take the exam.)

These problems and their proper solutions are important. Check with fellow students to verify that you (or they) have not made costly errors. There is NO PARTIAL CREDIT on any problem. Any error, no matter how insignificant is still an error.

1) $2\sin^2 x - 1 = -\sin x$

2) $2\cos^2 x = \sin x - 1$

3) $\sin^2 x = 1$

4) $\sin x = \tan x$

5) $\cot^2 x = \cot x$

6) $\sin x \cos x = 0$

7) $\sin^2 x = \sin x$

8) $2\sin x = 3$

9) $2\sin^2 x - \sqrt{3}\sin x = 0$

10) $3\tan x - 4 = \tan x - 2$

11) $2\sin^2 x = -\sin x$

12) $\tan x = \cot x$

13) $\sec^2 x = \frac{4}{3}$

14) $2\sin x \cos x = \sin x$

15) $2\cos x + 1 = 0$

16) $\cos x - 2\sin^2 x + 2 = 0$

17) $2\sin^2 x + \cos x = -\cos^2 x$

18) $\cos x \tan x = 3\tan x$

19) $2 + 2\sin x = 4\cos^2 x$

20) $4\cos^2 x - 3 = 0$

21) $\cos^2 x = \frac{1}{4}$

22) $2\sin x + 1 = 0$

23) $\sin x \cos x + \cos x - 1 - \sin x = 0$

24) $2\sin^2 - \sin x = 3$

25) $\sqrt{3}\tan x = -1$

26) $(3\sec x - 2)(\cos x) = 0$

27) $2\cos^2 = \cos x$

28) $\sin x + \sec^2 x = 1 + \sin^2 x + \tan^2 x$

29) $\cos^2 x - \sin^2 x = 1$

30) $\sin 2x = \cos x$

31) $\sin^2 \frac{x}{2} + \tan^2 \frac{x}{2} = \frac{3}{2}$

32) $\cos 2x = \sin x$

33) $2\sin^2 x - \cos x = 1$

34) $\sin 2x = 2\sin x$

35) $\sin 2x = 2\cos x$

36) $\sin(\sin^{-1} x) = x$

37) $\operatorname{Tan}(\operatorname{Tan}^{-1} x) = x$

38) $\operatorname{Sec}(\operatorname{Sec}^{-1} x) = x$

39) $\operatorname{Csc}(\operatorname{Csc}^{-1} x) = x$

40) $\operatorname{Tan}^{-1} x + \operatorname{Tan}^{-1} 1 = \frac{3\pi}{4}$

41) $2 \operatorname{Sin} x \operatorname{Sec} x = \operatorname{Sec} x$

42) $3 \operatorname{Tan} \frac{x}{2} = \operatorname{Cot} \frac{x}{2}$

43) $\operatorname{Sin}^2(x + \frac{\pi}{6}) - \operatorname{Cos}^2(x + \frac{\pi}{6}) = \frac{\sqrt{2}}{2}$

44) $\operatorname{Cos} 2x + 2 \operatorname{Sin}^2 \frac{x}{2} = 1$

45) $\operatorname{Csc}^2 x + 2 \operatorname{Csc} x = 0$

46) $\operatorname{Sec}^2 2x + \operatorname{Tan} 2x = 1$

47) $2 \operatorname{Sin}^2 x - 3 \operatorname{Sin} x + 1 = 0$

48) $2 \operatorname{Sin}^2 x + 3 \operatorname{Sin} x - 2 = 0$

49) $\operatorname{Cot} x \operatorname{Tan} 2x = 3$

50) $\operatorname{Sin}^{-1}(\operatorname{Sin} x) = x$

51) $\operatorname{Cos}^{-1}(\operatorname{Cos} x) = x$

52) $\operatorname{Tan}(\operatorname{Tan}^{-1} x) = x$

53) $\operatorname{Csc}^{-1}(\operatorname{Csc} x) = x$

54) $\operatorname{Cot}^{-1}(\operatorname{Cot} x) = x$

55) $2 \operatorname{Sec} x - 2\sqrt{2} = 0$

56) $2 \operatorname{Cot}^2 x + 2 \operatorname{Cot} x = 0$

57) $\operatorname{Sin} x + \operatorname{Cos} x = 1$

58) $\operatorname{Sin} 2x \operatorname{Sin} x - \operatorname{Cos} 2x \operatorname{Cos} x = 1$

59) $2 \operatorname{Sin}(x - \frac{\pi}{6}) = \sqrt{3} \operatorname{Sin} x$

60) $2 \operatorname{Sin}^2 x - \operatorname{Sin} x - 1 = 0$

61) $\operatorname{Sin}^2(x + \frac{\pi}{2}) + \operatorname{Cos}^2(x + \frac{\pi}{2}) - 1 = 0$

62) $2 \operatorname{Cos} 2x + \operatorname{Sin} 2x \operatorname{Sec} x = 0$

63) $\operatorname{Cos}(\operatorname{Cos}^{-1} x) = x$

64) $2 \operatorname{Cos}^2 x \operatorname{Sin}^2 x - \operatorname{Cos} x \operatorname{Sin} x = 0$

65) $3 \operatorname{Sec} x - \operatorname{Cos} x = 2$

66) $\operatorname{Sec}^{-1}(\operatorname{Sec} x) = x$

67) $2 \operatorname{Tan} x \operatorname{Sin} x + 2 \operatorname{Sin} x = \operatorname{Tan} x + 1$

68) $\operatorname{Sin}(\operatorname{Tan}^{-1} U) = x$

69) $\operatorname{Cot}(\operatorname{Cot}^{-1} x) = x$

70) $\operatorname{Sin} 2x \operatorname{Sin} x + \operatorname{Cos} 2x \operatorname{Cos} x = 1$

71) $\operatorname{Cos}(\operatorname{Tan}^{-1} U) = x$

72) $2 + \operatorname{Sec} x = 0$

73) $\operatorname{Tan}(\operatorname{Tan}^{-1} x + \operatorname{Tan}^{-1} 1) = -3$

74) $\operatorname{Tan}(\operatorname{Cos}^{-1} U) = x$

75) $\operatorname{Cos}(\operatorname{Sin}^{-1} U) = x$

76) $2 \operatorname{Cos}^2 x \operatorname{Sin}^2 x - \operatorname{Cos} x \operatorname{Sin} x = 0$

77) $\operatorname{Cos}(\frac{1}{2} \operatorname{ArcTan} \frac{3}{4}) = x$

78) $\operatorname{Sin}(\operatorname{Cos}^{-1} U) = x$

79) $\operatorname{Cos}\left[(\operatorname{ArcSin}(-\frac{1}{2}) + \operatorname{Cos}^{-1} \frac{5}{13}\right] = x$

80) $\operatorname{Tan}(\pi + \operatorname{ArcSin} \frac{2}{3}) = x$

81) $\operatorname{Sin}(\frac{1}{2} \operatorname{ArcTan} \frac{3}{5}) = x$

82) $\operatorname{Cot}^2 x - \operatorname{Cos}^2 x = \operatorname{Cot}^2 x \operatorname{Cos}^2 x$

83) $2 \operatorname{Cos} x = 4 \operatorname{Cos} x \operatorname{Csc} x$

84) $\operatorname{Tan} 2x = 3 \operatorname{Cot} 2x$

85) $\operatorname{Cos}^3 x - \operatorname{Sin}^3 x = (\operatorname{Cos} x - \operatorname{Sin} x)(1 + \frac{1}{2} \operatorname{Sin} 2x)$

86) $\operatorname{Cot}(\operatorname{Cos}^{-1} x) = x$

87) $\operatorname{Sin}(\operatorname{Sec}^{-1} x) = x$

88) $\operatorname{Sec}^{-1} \frac{5}{3} + \operatorname{Csc}^{-1} \frac{5}{4} = 2 \operatorname{Tan}^{-1} \frac{x}{3}$

89) $\operatorname{Tan}^2 x - \operatorname{Sec}^2 x = -1$

90) $\operatorname{Tan}(\operatorname{Csc}^{-1} U) = x$

91) $\operatorname{Sin}^{-1} \frac{3}{5} + \operatorname{Sin}^{-1} \frac{12}{13} = \operatorname{Cos}^{-1} \frac{-x}{65}$

92) $1 - 2 \operatorname{Sin}^2 x = \frac{1 - \operatorname{Tan}^2 x}{1 + \operatorname{Tan}^2 x}$

93) $\operatorname{Cos}(\operatorname{Csc}^{-1} U) =$

① $2\sin^2 x - 1 = -\sin x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1)$

$\sin x = \frac{1}{2} \quad \sin x = -1$

$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$

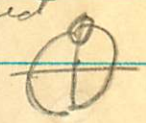
② $2\cos^2 x = \sin x - 1$

$2(1 - \sin^2 x) = \sin x - 1$

$2\sin^2 x + \sin x - 3 = 0$

$(2\sin x + 3)(\sin x - 1) = 0$

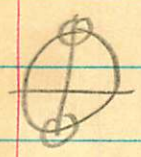
$\sin x = -\frac{3}{2}$ Reject $\quad \sin x = 1$



$x = \frac{\pi}{2}$

③ $\sin^2 x = 1$

$\sin x = \pm 1$

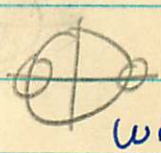


$x \in \{\frac{\pi}{2}, \frac{3\pi}{2}\}$

④ $\sin x = \tan x$

$\sin x(1 - \cos x) = 0$

$\sin x = 0 \quad \cos x = 1$



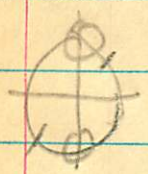
$x \in \{0, \pi\}$

WRONG ANS $0, \frac{\pi}{2}, \pi$

⑤ $\cot^2 x = \cot x$

$\cot x(\cot x - 1) = 0$

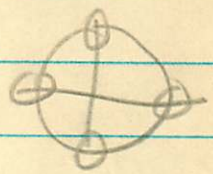
$\cot x = 0 \quad \cot x = 1$



$x \in \{\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}\}$

⑥ $\sin x \cos x = 0$

$\sin x = 0 \quad \cos x = 0$

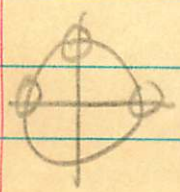


$x \in \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$

⑦ $\sin^2 x = \sin x$

$\sin x(\sin x - 1) = 0$

$\sin x = 0 \quad \sin x = 1$



$x \in \{0, \frac{\pi}{2}, \pi\}$

⑧ $2\sin x = 3$

$\sin x = \frac{3}{2}$

Reject $\quad \phi$

(9) $2\sin^2 x - \sqrt{3}\sin x = 0$

$\sin x(2\sin x - \sqrt{3}) = 0$

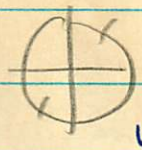
$\sin x = 0 \quad \sin x = \frac{\sqrt{3}}{2}$

$x = \{0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi\}$

(10) $3\tan x - 4 = \tan x - 2$

$2\tan x = 2$

$\tan x = 1$



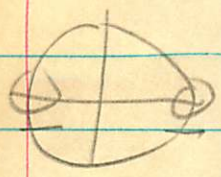
$x = \{\frac{\pi}{4}, \frac{5\pi}{4}\}$

Wrong ans $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(11) $2\sin^2 x = -\sin x$

$\sin x(2\sin x + 1) = 0$

$\sin x = 0 \quad \sin x = -\frac{1}{2}$

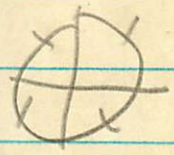


$x = \{0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

(12) $\tan x = \cot x$

$\tan^2 x = 1$

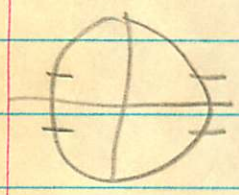
$\tan x = \pm 1$



$x = \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$

(13) $\sec^2 x = \frac{4}{3}$

$\sec x = \pm \frac{2}{\sqrt{3}}$

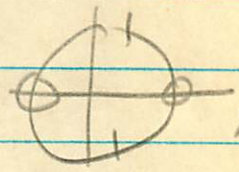


$x = \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

(14) $2\sin x \cos x = \sin x$

$\sin x(2\cos x - 1) = 0$

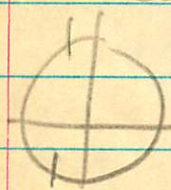
$\sin x = 0 \quad \cos x = \frac{1}{2}$



$x = \{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\}$

(15) $2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$



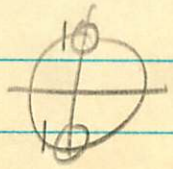
$x = \{\frac{2\pi}{3}, \frac{4\pi}{3}\}$

(16) $\cos x - 2\sin^2 x + 2 = 0$

$2\cos^2 x + \cos x = 0$

$\cos x(2\cos x + 1) = 0$

$\cos x = 0 \quad \cos x = -\frac{1}{2}$



$x = \{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\}$

Wrong ans $\frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$

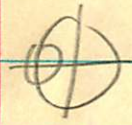
(17) $2\sin^2 x + \cos x = -\cos^2 x$

$2 - 2\cos^2 x + \cos x = -\cos^2 x$

$\cos^2 x - \cos x - 2 = 0$

$(\cos x - 2)(\cos x + 1) = 0$

Reject $\cos x = -1$

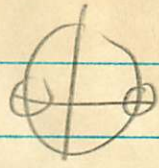


$x = \pi$

(18) $\cos x \tan x = 3\tan x$

$\tan x(\cos x - 3) = 0$

$\tan x = 0$ Reject



$x \in \{0, \pi\}$

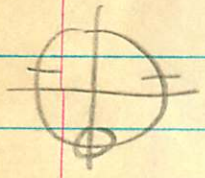
(19) $2 + 2\sin x = 4\cos^2 x$

$1 + \sin x = 2 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

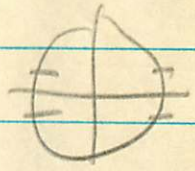
$\sin x = \frac{1}{2}$ $\sin x = -1$



$x \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

(20) $4\cos^2 x - 3 = 0$

$\cos x = \pm\frac{\sqrt{3}}{2}$



$x \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

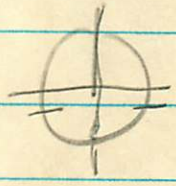
(21) $\cos^2 x = \frac{1}{4}$

$\cos x = \pm\frac{1}{2}$

$x \in \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

(22) $2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$



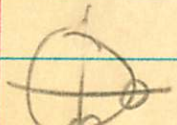
$x \in \left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

(23) $\sin x \cos x + \cos x - 1 - \sin x = 0$

$\sin x(\cos x - 1) + (\cos x - 1) = 0$

$(\cos x - 1)(\sin x + 1) = 0$

$\cos x = 1$ $\sin x = -1$



$x \in \left\{0, \frac{3\pi}{2}\right\}$

(24) $2\sin^2 x - \sin x = 3$

$2\sin^2 x - \sin x - 3 = 0$

$(2\sin x - 3)(\sin x + 1) = 0$

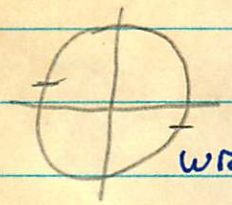
$\sin x = \frac{3}{2}$ $\sin x = -1$

reject

$x = \frac{3\pi}{2}$

(25) $\sqrt{3}\tan x = -1$

$\tan x = -\frac{1}{\sqrt{3}} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$



$x \in \left\{\frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

WRong $\frac{\pi}{6}, \frac{7\pi}{6}$

(26) $(3\sec x - 2)(\cos x) = 0$

$\sec x = \frac{2}{3}$ $\cos x = 0$

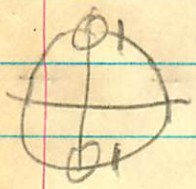
Reject Reject since $\sec x$ in Equation $\cos x \neq 0$.

ϕ

(27) $2\cos^2 x = \cos x$

$\cos x(2\cos x - 1) = 0$

$\cos x = 0$ $\cos x = \frac{1}{2}$



$x \in \left\{\frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}\right\}$

(28) $\sin x + \sec^2 x = 1 + \sin^2 x + \tan^2 x$

$s^2 + c^2 = 1$

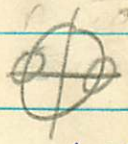
$t^2 + 1 = \sec^2$ so subtract

$\sin x = \sin^2 x$

$\sin x(\sin x - 1) = 0$

$\sin x = 0$ $\sin x = 1$

$x = \frac{\pi}{2}$ Reject



$x \in \{0, \pi\}$

WRong answers $\{0, \frac{\pi}{2}, \pi\}$

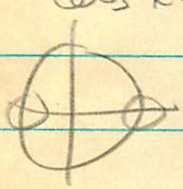
(29) $\cos^2 x - \sin^2 x = 1$

$\cos^2 x - 1 + \cos^2 x = 1$

$2\cos^2 x = 2$

$\cos^2 x = 1$

$\cos x = \pm 1$



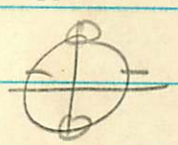
$x \in \{0, \pi\}$

(30) $\sin 2x = \cos x$

$2\sin x \cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ $\sin x = \frac{1}{2}$



$x \in \left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

(31) $\sin^2 \frac{x}{2} + \tan^2 \frac{x}{2} = \frac{3}{2}$

$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2$

$1 - \cos^2 \frac{x}{2} + \sec^2 \frac{x}{2} - 1 = \frac{3}{2}$

$2\sec^2 \frac{x}{2} - 2\cos^2 \frac{x}{2} = 3$

$2 - 2\cos^4 \frac{x}{2} = 3\cos^2 \frac{x}{2}$

$2\cos^4 \frac{x}{2} + 3\cos^2 \frac{x}{2} - 2 = 0$

$(2\cos^2 \frac{x}{2} - 1)(\cos^2 \frac{x}{2} + 2) = 0$

$\cos^2 \frac{x}{2} = +\frac{1}{2}$

$\cos \frac{x}{2} = \pm \frac{\sqrt{2}}{2}$

$0 \leq x < 2\pi$

$0 \leq \frac{x}{2} < \pi$

$\frac{x}{2} \in \left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

$x \in \left\{\frac{\pi}{2}, \frac{5\pi}{2}\right\}$

(32) $\cos 2x = \sin x$

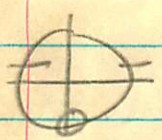
$\cos^2 x - \sin^2 x = \sin x$

$1 - 2\sin^2 x = \sin x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1)$

$\sin x = \frac{1}{2} \quad \sin x = -1$



$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$

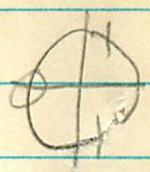
(33) $2\sin^2 x - \cos x = 1$

$2 - 2\cos^2 x - \cos x = 1$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2} \quad \cos x = -1$



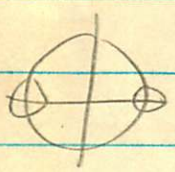
$x \in \{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\}$

(34) $\sin 2x = 2\sin x$

$2\sin x \cos x - 2\sin x = 0$

$2\sin x(\cos x - 1) = 0$

$\sin x = 0 \quad \cos x = 1$



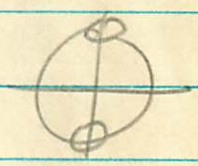
$x \in \{0, \pi\}$

(35) $\sin 2x = 2\cos x$

$2\sin x \cos x - 2\cos x = 0$

$2\cos x(\sin x - 1) = 0$

$\cos x = 0 \quad \sin x = 1$



$x \in \{\frac{\pi}{2}, \frac{3\pi}{2}\}$

(36) $\sin(\sin^{-1} x) = x$

$|x| \leq 1$

(37) $\tan(\tan^{-1} x) = x$

$x \in \mathbb{R}$

Wrong Ans $-\frac{\pi}{2} < x < \frac{\pi}{2}$

(38) $\sec(\sec^{-1} x) = x$

$|x| \geq 1$

(39) $\csc(\csc^{-1} x) = x$

$|x| \geq 1$

(40) $\tan^{-1} x + \tan^{-1} 1 = \frac{3\pi}{4}$

$\tan^{-1} x + \frac{\pi}{4} = \frac{3\pi}{4}$

$\tan^{-1} x = \frac{\pi}{2}$

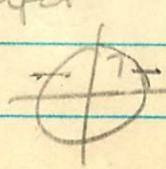
Reject $\tan^{-1} x \neq \frac{\pi}{2}$

(1) Wrong ans $\frac{\pi}{2}$

(41) $2\sin x \sec x = \sec x$

$\sec x(2\sin x - 1) = 0$

Reject $\sin x = \frac{1}{2}$



$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}\}$

(42) $3\tan\frac{x}{2} = \cot\frac{x}{2}$

$0 \le x < 2\pi$

$0 \le \frac{x}{2} < \pi$

$\tan^2\frac{x}{2} = \frac{1}{3}$

$\tan\frac{x}{2} = \pm\frac{1}{\sqrt{3}}$

$\frac{x}{2} \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

$x \in \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

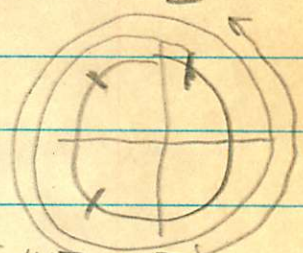
(43) $\sin^2\left(x+\frac{\pi}{6}\right) - \cos^2\left(x+\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$

$\cos 2\left(x+\frac{\pi}{6}\right) = -\frac{\sqrt{2}}{2}$

$0 < x < 2\pi$

$\frac{\pi}{6} < x+\frac{\pi}{6} < 2\pi+\frac{\pi}{6}$

$\frac{\pi}{3} < 2\left(x+\frac{\pi}{6}\right) < 4\pi+\frac{\pi}{3}$



$2\left(x+\frac{\pi}{6}\right) \in \left\{\frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}\right\}$

$\div 2$ $x+\frac{\pi}{6} \in \left\{\frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}\right\}$

$x \in \left\{\frac{5\pi}{24}, \frac{11\pi}{24}, \frac{29\pi}{24}, \frac{35\pi}{24}\right\}$

(44) $\cos 2x + 2\sin^2\frac{x}{2} = 1$

$\cos^2 x - \sin^2 x + 2\sin^2\frac{x}{2} = 1$

$1 - 2\sin^2 x + 2\sin^2\frac{x}{2} = 1$

$-\sin^2 x + \sin^2\frac{x}{2} = 0$

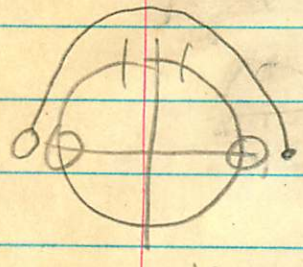
$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$-(4\sin^2\frac{x}{2}\cos^2\frac{x}{2}) + \sin^2\frac{x}{2} = 0$

$-\sin^2\frac{x}{2}\left(4\cos^2\frac{x}{2} - 1\right) = 0$

$\sin^2\frac{x}{2} = 0$ $\cos^2\frac{x}{2} = \frac{1}{4}$

$\cos^2\frac{x}{2} = \pm\frac{1}{2}$



$0 \le x < 2\pi$

$0 \le \frac{x}{2} < 2\pi$

$\frac{x}{2} \in \left\{0, \frac{\pi}{3}, \frac{2\pi}{3}\right\}$

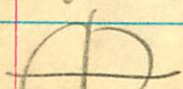
$x \in \left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

(45) $\csc^2 x + 2\csc x = 0$

$\csc x(\csc x + 2) = 0$

$\csc x = 0$ $\csc x = -2$

Reject $\sin x = -\frac{1}{2}$



$x \in \left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

(46) $\sec^2 2x + \tan 2x = 1$

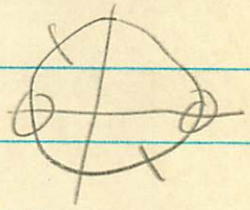
$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2$

$\tan^2 2x + 1 + \tan 2x = 1$

$\tan 2x(\tan 2x + 1) = 0$

$\tan 2x = 0$ $\tan 2x = -1$



$0 \le x < 2\pi$

$0 \le 2x < 4\pi$

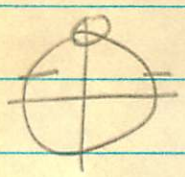
$2x \in \left\{0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi, \frac{11\pi}{4}, 3\pi, \frac{15\pi}{4}\right\}$

$x \in \left\{0, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{7\pi}{8}, \pi, \frac{11\pi}{8}, \frac{3\pi}{2}, \frac{15\pi}{8}\right\}$

(47) $2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \frac{1}{2} \quad \sin x = 1$

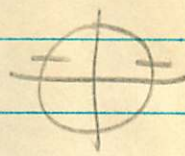


$x \in \{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}\}$

(48) $2\sin^2 x + 3\sin x - 2 = 0$

$(2\sin x - 1)(\sin x + 2)$

$\sin x = \frac{1}{2} \quad \sin x = -2$ Reject



$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}\}$

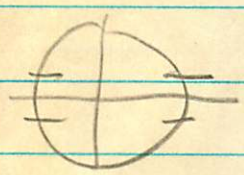
(49) $\cot x \tan 2x = 3$

$\frac{\cot x \cdot 2\tan x}{1 - \tan^2 x} = 3$

$2 = 3 - 3\tan^2 x$

$3\tan^2 x = 1$

$\tan x = \pm \frac{1}{\sqrt{3}}$



$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

(50) $\sin^{-1}(\sin x) = x$

$|x| \leq \frac{\pi}{2}$

(51) $\cos^{-1}(\cos x) = x$

$0 \leq x \leq \pi$

(52) $\tan(\tan^{-1} x) = x$

$x \in \mathbb{R}$

(53) $\csc^{-1}(\csc x) = x$

$|x| \leq \frac{\pi}{2}, \; x \neq 0$

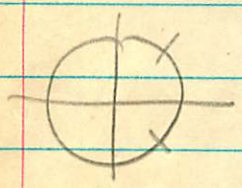
(54) $\cot^{-1}(\cot x) = x$

$0 < x < \pi$

(55) $2\sec x - 2\sqrt{2} = 0$

$\sec x = \sqrt{2}$

$\cos x = \frac{\sqrt{2}}{2}$

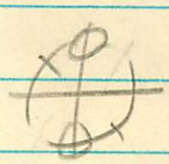


$x \in \{\frac{\pi}{4}, \frac{7\pi}{4}\}$

(56) $2\cot^2 x + 2\cot x = 0$

$2\cot x(\cot x + 1) = 0$

$\cot x = 0 \quad \cot x = -1$



$x \in \{\frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\}$

(57) $\sin x + \cos x = 1$

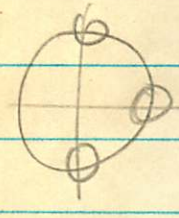
$\sin^2 x = 1 - 2\cos x + \cos^2 x$

$1 - \cos^2 x =$

$2\cos^2 x - 2\cos x = 0$

$2\cos x(\cos x - 1) = 0$

$\cos x = 0 \quad \cos x = 1$



$x \in \{0, \frac{\pi}{2}, \frac{3\pi}{2}\}$

$\sin x + \cos x = 1$

$0 \quad 0 + 1 = 1$ ok

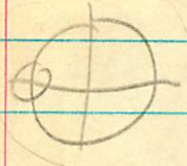
$\frac{\pi}{2} \quad 1 + 0 = 1$ ok

$\frac{3\pi}{2} \quad -1 + 0 = 1$ NO

$x \in \{0, \frac{\pi}{2}\}$

(58) $\sin 2x \sin x - \cos 2x \cos x = 1$

$\cos 3x = -1$



$3x \in \{\pi, 3\pi, 5\pi\}$

$x \in \{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\}$

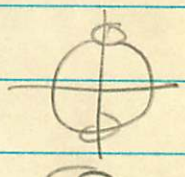
(59) $2\sin(x - \frac{\pi}{6}) = \sqrt{3}\sin x$

$2(\sin x \cos\frac{\pi}{6} - \sin\frac{\pi}{6}\cos x) =$

$2(\sin x(\frac{\sqrt{3}}{2}) - \frac{1}{2}\cos x) =$

$\sqrt{3}\sin x = \cos x > \sqrt{3}\sin x$

$\cos x = 0$

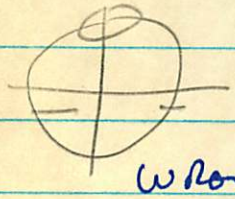


$x \in \{\frac{\pi}{2}, \frac{3\pi}{2}\}$

(60) $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2} \quad \sin x = 1$



$x \in \{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

wrong ans $\frac{\pi}{6}$ $\frac{5\pi}{6}$ $\frac{3\pi}{2}$

wrong answer include $\frac{\pi}{2}$

Reject $\frac{\pi}{2}$

because Secant in the Problem.

$x \in \{\frac{7\pi}{6}, \frac{11\pi}{6}\}$

(61) $\sin^2(x + \frac{\pi}{2}) + \cos^2(x + \frac{\pi}{2}) - 1 = 0$

Hestom identity

$x \in \mathbb{R}$

(62) $2\cos 2x + \sin 2x \sec x = 0$

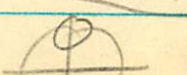
$2(\cos^2 x - \sin^2 x) + 2\sin x = 0$

$1 - 2\sin^2 x + \sin x = 0$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

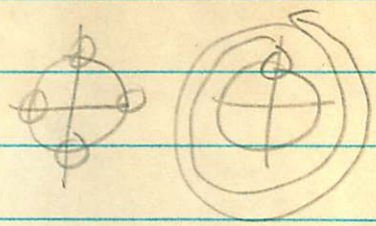
$\sin x = -\frac{1}{2} \quad \sin x = 1$



$x \in \{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

(63) $\cos(\cos^{-1}x) = x$
$|x| \le 1$

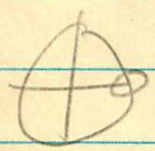
(64) $2\cos^2 x \sin^2 x - \cos x \sin x = 0$
$\sin x \cos x (2\sin x \cos x - 1) = 0$
$\sin x = 0 \quad \cos x = 0 \quad \sin 2x = 1$



$2x \in \{\frac{\pi}{2}, \frac{5\pi}{2}\}$
$x \in \{\frac{\pi}{4}, \frac{5\pi}{4}\}$

$x \in \{0, \frac{\pi}{4}, \frac{\pi}{2}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}\}$

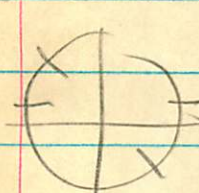
(65) $3\sec x - \cos x = 2$
$3 - \cos^2 x = 2\cos x$
$\cos^2 x + 2\cos x - 3 = 0$
$(\cos x - 1)(\cos x + 3) = 0$
$\cos x = 1 \quad \cos x = -3$ reject



$x = 0$

(66) $\sec^{-1}(\sec x) = x$
$0 \le x \le \pi; \; x \ne \frac{\pi}{2}$

(67) $2\tan x \sin x + 2\sin x = \tan x + 1$
$2\sin x(\tan x + 1) - (\tan x + 1) = 0$
$(\tan x + 1)(2\sin x - 1) = 0$
$\tan x = -1 \quad \sin x = \frac{1}{2}$

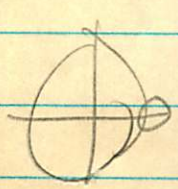


$x \in \{\frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{4}\}$

(68) $\sin(\tan^{-1}u) = x$
$x = \frac{u}{\sqrt{1+u^2}}$

(69) $\cot(\cot^{-1}x) = x$
$x \in \mathbb{R}$

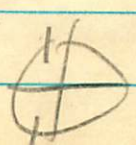
(70) $\sin 2x \sin x + \cos 2x \cos x = 1$
$\cos(2x - x) = 1$
$\cos x = 1$



$x = 0$

(71) $\cos(\tan^{-1}u) = x$
$x = \frac{1}{\sqrt{1+u^2}}$

(72) $2 + \sec x = 0$
$\sec x = -2$
$\cos x = -\frac{1}{2}$



$x \in \{\frac{2\pi}{3}, \frac{4\pi}{3}\}$

(81) $\sin\left(\frac{1}{2}\arctan\frac{3}{5}\right) = x$

$\sin\frac{1}{2}x = \pm\sqrt{\frac{1}{2}(1-\cos x)}$

$\cos\left(\tan^{-1}\frac{3}{5}\right) = \frac{1}{\sqrt{1+\frac{9}{25}}} = \frac{5}{\sqrt{34}}$

$= \pm\sqrt{\frac{1}{2}\left(1-\frac{5}{\sqrt{34}}\right)}$

$\pm\sqrt{\frac{34-5\sqrt{34}}{68}}$

(82) $\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$

$\frac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x} = \frac{\cos^4 x}{\sin^2 x}$

$x \in \mathbb{R}$

$x \neq \{k\pi\}$

$\frac{\cos^2 x(1-\sin^2 x)}{\sin^2 x}$

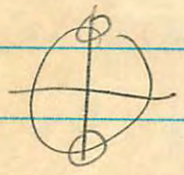
$\frac{\cos^4 x}{\sin^2 x} = \frac{\cos^4 x}{\sin^2 x}$

(83) $2\cos x = 4\cos x \csc x$

$2\cos x(1 - 2\csc x) = 0$

$\cos x = 0 \qquad \csc x = \frac{1}{2}$

Reject

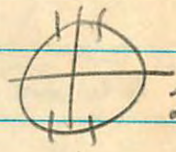


$x \in \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

(84) $\tan 2x = 3\cot 2x$

$\tan^2 2x = 3$

$\tan 2x = \pm\sqrt{3} = \pm\frac{\sqrt{3}}{1}$



$2x \in \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}\right\}$

$x \in \left\{\frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}\right\}$

(85) $\cos^3 x - \sin^3 x = (\cos x - \sin x)\left(1 + \frac{1}{2}\sin 2x\right)$

$(\cos x - \sin x)(\cos^2 x - \sin^2 x\cos^2 x + \sin^2 x) =$

$\cos x = \sin x \quad (\cos x - \sin x)(1 - \sin^2 x\cos^2 x) =$

$1 - \sin^2 x\cos^2 x = 1 + \sin x\cos x$

$\sin x\cos x(-\sin x\cos x - 1) = 0$

$\sin x\cos x = 1$

$\sin^2 x(1-\sin^2 x) = 1$

$\sin^2 x - \sin^4 x = 1$

Domain factor

$x \in \left\{0, \frac{\pi}{4}, \frac{\pi}{2}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}\right\}$

$(4x-9)(x-4)$

$x = \frac{9}{4} \qquad x = 4.$

(81) $\sin\left(\frac{1}{2}\arctan\frac{3}{5}\right)=x$

$\sin\frac{1}{2}x=\pm\sqrt{\frac{1}{2}(1-\cos x)}$

$\cos\left(\tan^{-1}\frac{3}{5}\right)=\frac{1}{\sqrt{1+\frac{9}{25}}}=\frac{5}{\sqrt{34}}$

$=\pm\sqrt{\frac{1}{2}\left(1-\frac{5}{\sqrt{34}}\right)}$

$\pm\sqrt{\frac{34-5\sqrt{34}}{68}}$

(82) $\cot^2x-\cos^2x=\cot^2x\cos^2x$

$\frac{\cos^2x-\cos^2x\sin^2x}{\sin^2x}=\frac{\cos^4x}{\sin^2x}$

$x\in\mathbb{R}$

$x\notin\{k\pi\}$

$\frac{\cos^2x(1-\sin^2x)}{\sin^2x}$

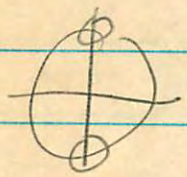
$\frac{\cos^4x}{\sin^2x}=\frac{\cos^4x}{\sin^2x}$

(83) $2\cos x=4\cos x\csc x$

$2\cos x(1-2\csc x)=0$

$\cos x=0$ $\quad\csc x=\frac{1}{2}$

Reject



$x\in\left\{\frac{\pi}{2},\frac{3\pi}{2}\right\}$

(84) $\tan 2x=3\cot 2x$

$\tan^2 2x=3$

$\tan 2x=\pm\sqrt{3}=\pm\frac{\sqrt{3}}{1}$

$2x\in\left\{\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3},\frac{8\pi}{3},\frac{10\pi}{3},\frac{11\pi}{3}\right\}$

$x\in\left\{\frac{\pi}{6},\frac{\pi}{3},\frac{2\pi}{3},\frac{5\pi}{6},\frac{7\pi}{6},\frac{4\pi}{3},\frac{5\pi}{3},\frac{11\pi}{6}\right\}$

(85) $\cos^3x-\sin^3x=(\cos x-\sin x)\left(1+\frac{1}{2}\sin 2x\right)$

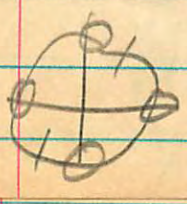
$(\cos x-\sin x)(\cos^2x-\sin^2x\cos^2x+\sin^2x)=$

$\cos x=\sin x\quad(\cos x-\sin x)(1-\sin^2x\cos^2x)=$

$1-\sin^2x\cos^2x=1+\sin x\cos x$

$\sin x\cos x(-\sin x\cos x-1)=0$

Don't factor



$\sin x\cos x=1$

$\sin^2x(1-\sin^2x)=1$

$\sin^2x-\sin^4x=1$

$x\in\left\{0,\frac{\pi}{4},\frac{\pi}{2},\pi,\frac{5\pi}{4},\frac{3\pi}{2}\right\}$

(81) $\sin\left(\frac{1}{2}\arctan\frac{3}{5}\right)=x$

$\sin\frac{1}{2}x=\pm\sqrt{\frac{1}{2}(1-\cos x)}$

$\cos\left(\tan^{-1}\frac{3}{5}\right)=\frac{1}{\sqrt{1+\frac{9}{25}}}=\frac{5}{\sqrt{34}}$

$=\pm\sqrt{\frac{1}{2}\left(1-\frac{5}{\sqrt{34}}\right)}$

$\pm\sqrt{\frac{34-5\sqrt{34}}{68}}$

(82) $\cot^2x-\cos^2x=\cot^2x\cos^2x$

$\frac{\cos^2x-\cos^2x\sin^2x}{\sin^2x}=\frac{\cos^4x}{\sin^2x}$

$x\in\mathbb{R}$

$x\neq\{k\pi\}$

$\frac{\cos^2x(1-\sin^2x)}{\sin^2x}$

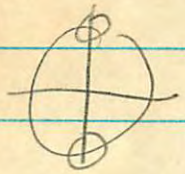
$\frac{\cos^4x}{\sin^2x}=\frac{\cos^4x}{\sin^2x}$

(83) $2\cos x=4\cos x\csc x$

$2\cos x(1-2\csc x)=0$

$\cos x=0$ $\quad\csc x=\frac{1}{2}$

Reject

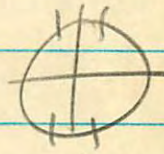


$x\in\left\{\frac{\pi}{2},\frac{3\pi}{2}\right\}$

(84) $\tan 2x=3\cot 2x$

$\tan^2 2x=3$

$\tan 2x=\pm\sqrt{3}=\pm\frac{\sqrt{3}}{1}$



$2x\in\left\{\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3},\frac{8\pi}{3},\frac{10\pi}{3},\frac{11\pi}{3}\right\}$

$x\in\left\{\frac{\pi}{6},\frac{\pi}{3},\frac{2\pi}{3},\frac{5\pi}{6},\frac{7\pi}{6},\frac{4\pi}{3},\frac{5\pi}{3},\frac{11\pi}{6}\right\}$

(85) $\cos^3x-\sin^3x=(\cos x-\sin x)(1+\frac{1}{2}\sin 2x)$

$(\cos x-\sin x)(\cos^2x-\sin^2x\cos^2x+\sin^2x)=$

$\cos x=\sin x(\cos x-\sin x)(1-\sin^2x\cos^2x)=$

$1-\sin^2x\cos^2x=1+\sin x\cos x$

$\sin x\cos x(-\sin x\cos x-1)=0$

$\sin x\cos x=1$

$\sin^2x(1-\sin^2x)=1$

$\sin^2x-\sin^4x=1$

Don't factor

$x\in\left\{0,\frac{\pi}{4},\frac{\pi}{2},\pi,\frac{5\pi}{4},\frac{3\pi}{2}\right\}$

(86) $\cot(\cos^{-1}x) = x$

$\frac{\cos(\cos^{-1}x)}{\sin(\cos^{-1}x)} =$

$\frac{x}{\sqrt{1-x^2}} = x$

$1 = \sqrt{1-x^2}$

$1 = 1-x^2$

$x = 0$

(87) $\sin(\sec^{-1}u) = x$

$\sin(\cos^{-1}(\frac{1}{u})) = x$

$\sqrt{1-\frac{1}{u^2}} = x$

$\frac{\sqrt{u^2-1}}{u} = x$

(88) $\sec^{-1}\frac{5}{3} + \csc^{-1}\frac{5}{4} = 2\tan^{-1}\frac{x}{3}$

$\sin\left(\cos^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}\right) = \sin\left(2\tan^{-1}\frac{x}{3}\right)$

$\sin\cos^{-1}\frac{3}{5}\cos\sin^{-1}\frac{4}{5} + \frac{4}{5}\cdot\frac{3}{5} = 2\sin\tan^{-1}\frac{x}{3}\cos\tan^{-1}\frac{x}{3}$

$\sqrt{1-\frac{9}{25}}\sqrt{1-\frac{16}{25}} + \frac{12}{25} = 2\frac{\frac{x}{3}}{\sqrt{1+\frac{x^2}{9}}}\cdot\frac{1}{\sqrt{1+\frac{x^2}{9}}}$

$\frac{4}{5}\cdot\frac{3}{5} + \frac{12}{25}$

$\frac{24}{25} = \frac{\frac{2x}{3}}{\frac{9+x^2}{3}}$

$\frac{24}{25} = \frac{6x}{9+x^2}$

$\frac{4}{25} = \frac{x}{9+x^2}$

$4x^2 + 36 = 25x$

$4x^2 - 25x + 36 = 0$

$(4x-9)(x-4)$

$x = \frac{9}{4} \quad x = 4.$

$x \in \{\frac{9}{4}, 4\}$

(89) $\tan^2 x - \sec^2 x = -1$

$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2$

$\tan^2 x - (\tan^2 x + 1) = -1$

$-1 = -1$

$x \in \mathbb{R} \quad x \neq \left\{\frac{(2k+1)\pi}{2}\right\}$

(90) $\tan(\csc^{-1} u) = x$

$$\frac{\sin\left(\sin^{-1}\left(\frac{1}{u}\right)\right)}{\cos\left(\sin^{-1}\left(\frac{1}{u}\right)\right)} = x$$

$$\frac{\frac{1}{u}}{\sqrt{1-\frac{1}{u^2}}} =$$

$$\frac{\frac{1}{u}}{\frac{\sqrt{u^2-1}}{u}} = \frac{1}{\sqrt{u^2-1}}$$

(91) $\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{12}{13} = \cos^{-1}\frac{-x}{65}\right)$

$$\cos\sin^{-1}\frac{3}{5}\cos\sin^{-1}\frac{12}{13} - \frac{3}{5}\cdot\frac{12}{13} = \frac{-x}{65}$$

$$\sqrt{1+\frac{9}{25}}\sqrt{1-\frac{144}{169}} - \frac{36}{65} = \frac{-x}{65}$$

$$\sqrt{\frac{16}{25}}\sqrt{\frac{25}{169}} - \frac{36}{65}$$

$$\frac{4}{5}\cdot\frac{5}{13} - \frac{36}{65} = \frac{-x}{65}$$

$$\frac{20}{65} - \frac{36}{65}$$

$$\frac{-16}{65} = \frac{-x}{65}$$

$$x = 16$$

(92) $1 - 2\sin^2 x = \frac{1-\tan^2 x}{1+\tan^2 x}$

$$= \frac{1-\frac{\sin^2 x}{\cos^2 x}}{1+\frac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$$

$1 - 2\sin^2 x = 1 - 2\sin^2 x$

Identity.

$x \in \mathbb{R}, \; x \neq \left\{\frac{(2k+1)\pi}{2}\right\}$

(93) $\cos(\csc^{-1} u) = x$

$\cos\left(\sin^{-1}\left(\frac{1}{u}\right)\right) = x$

$$\sqrt{1-\frac{1}{u^2}} =$$

$$\frac{\sqrt{u^2-1}}{u} = x$$

15 minutes.

18 ↑ +
15–17 ✓
12–14 0
9–11 −
<9 =

I For what values of u are these true?
12 poss

a) $\cos(\cos^{-1} u) = u$
$-1 \le u \le 1$

b) $\cos^{-1}(\cos u) = u$
$0 \le u \le \pi$

c) $\sin(\sin^{-1} u) = u$
$-1 \le u \le 1$

d) $\sin^{-1}(\sin u) = u$
$-\frac{\pi}{2} \le u \le \frac{\pi}{2}$

e) $\tan(\tan^{-1} u) = u$
$u \in \mathbb{R}$

f) $\tan^{-1}(\tan u) = u$
$-\frac{\pi}{2} < u < \frac{\pi}{2}$

g) $\sec(\sec^{-1} u) = u$
$|u| \ge 1$

h) $\sec^{-1}(\sec u) = u$
$0 \le u \le \pi;\ u \ne \frac{\pi}{2}$

i) $\csc(\csc^{-1} u) = u$
$|u| \ge 1$

j) $\csc^{-1}(\csc u) = u$
$-\frac{\pi}{2} \le u \le \frac{\pi}{2};\ u \ne 0$

k) $\cot(\cot^{-1} u) = u$
$u \in \mathbb{R}$

l) $\cot^{-1}(\cot u) = u$
$0 < u < \pi$

II Prove $\cos(\tan^{-1} u) = \frac{1}{\sqrt{1+u^2}}$
9 poss

let $\tan^{-1} u \overset{①}{=} b$ $\quad \overset{②}{-\frac{\pi}{2} < b < \frac{\pi}{2}}$ $\quad \overset{③}{u \in \mathbb{R}}$

$\tan(\tan^{-1} u) = \tan b$

$u = \tan b$

$u \cos b = \sin b$

④ $u^2 \cos^2 b = \sin^2 b$

$u^2 \cos^2 b = 1 - \cos^2 b$

$u^2 \cos^2 b + \cos^2 b = 1$

$\cos^2 b\,(u^2+1) = 1$

$\cos^2 b \overset{⑤}{=} \frac{1}{u^2+1}$

$\cos b \overset{⑥}{=} \frac{\pm 1}{\sqrt{u^2+1}}$

$\cos \tan^{-1} u \overset{⑦}{=} \frac{1}{\sqrt{u^2+1}}$

(8, 9) b is element in 4th or 1st quad and cosine of any thing in that interval is positive

Quiz #2 chap 4

Given after collections of HW #5.

15 minutes

Emphasis on $\pm$ & $\frac{1}{1}$ how you get rid of it

I Prove $\cos(\tan^{-1} u) = \dfrac{1}{\sqrt{1+u^2}}$

① Let $\tan^{-1} u = b$ $u \in \mathbb{R}$ ② $|b| < \frac{\pi}{2}$ ③

$u = \tan b$

$u \cos b = \sin b$

$u^2 \cos^2 b = \sin^2 b$

④ $u^2 \cos^2 b = 1 - \cos^2 b$

$(u^2+1)\cos^2 b = 1$ ⑤

$\cos b = \dfrac{\pm 1}{\sqrt{u^2+1}}$

$b \in$ 1st & 4th Quad.

⑥,⑦ cos of anything in 1st & 4th Quad is > 0

$\cos(\tan^{-1} u) = \dfrac{1}{\sqrt{1+u^2}}$ ⑧

Scoring 17↑ +

14-16 ✓

10-13 0

7-9 −

0-6 =

II Prove $\cos(\sin^{-1} u) = \sin(\cos^{-1} u)$

$= \sqrt{1-u^2}$

Let ① $\sin^{-1} u = b$ ② $|u| \le 1$ ③ $|b| \le \frac{\pi}{2}$

$u = \sin b$

④ $u^2 = 1 - \cos^2 b$

$\cos^2 b = 1 - u^2$

⑤ $\cos b = \pm\sqrt{1-u^2}$

$b \in$ 1st & 4th Quad.

cos of anything in 1st & 4th Quad > 0

⑥ $\cos(\sin^{-1} u) = \sqrt{1-u^2}$

Let ⑦ $\cos^{-1} u = b$ ⑧ $|u| \le 1$ ⑨ $0 \le b \le \pi$

$u = \cos b$

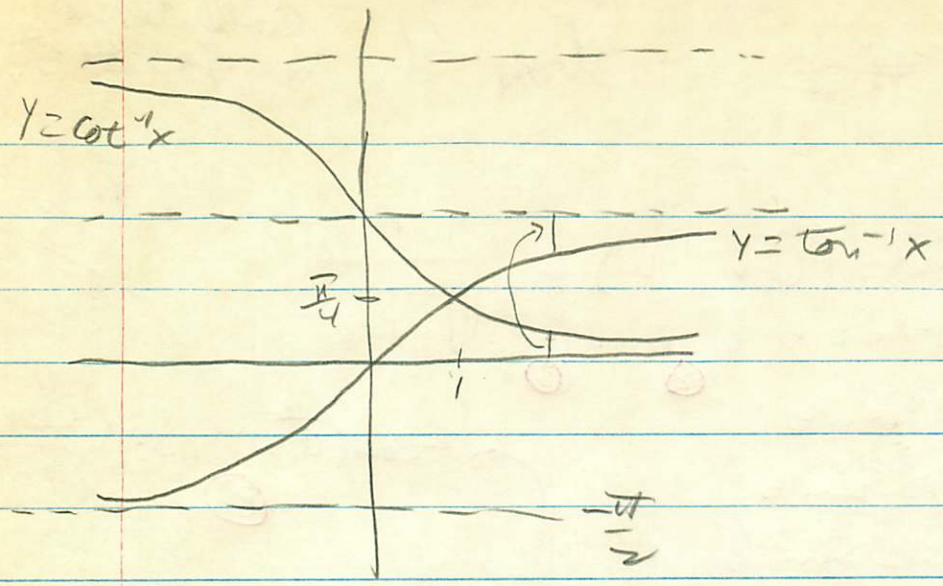
⑩ $u^2 = 1 - \sin^2 b$

$\sin^2 b = 1 - u^2$

⑪ $\sin b = \pm\sqrt{1-u^2}$

$b \in$ 1st & 2nd Quad

⑫ sin of anything in 1st, 2nd Quad > 0

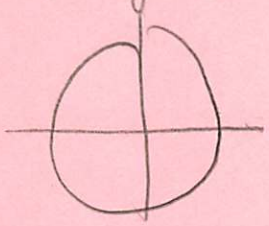


Main reason the "Table" function has $\frac{\pi}{2}$ for $\tan^{-1} x + \cot^{-1} x$

Lecture #5 Lecture DAY after Freedom Day is redundant.

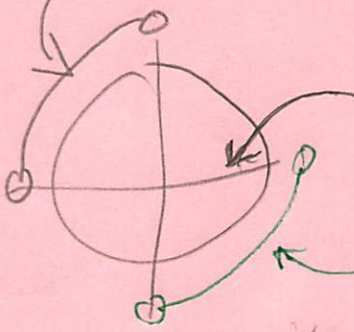
WTP $\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right) + \pi$ for $x < 0$

Let $b = \cot^{-1} x$ $\quad 0 < b < \pi$ but (range of $\cot^{-1} x$)



$\cot b = x$

$\tan b = \frac{1}{x}$



when $x<0$ then $\frac{\pi}{2} < b < \pi$

"b" in radian measurement

$\tan^{-1}(\tan b) = \tan^{-1}\left(\frac{1}{x}\right)$

this is NOT = to b because the range of $\tan^{-1} x$ is $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$

and since

Since tangent has period π,

$\tan b = \tan(b+\pi) = \tan(b-\pi)$

$\frac{\pi}{2} < b < \pi$ $\quad \frac{3\pi}{2} + \frac{3\pi}{2}$

this is just one we want

$-\frac{\pi}{2} < \quad < \frac{\pi}{2}$

So $\tan^{-1}\tan(b-\pi) = \tan\left(\frac{1}{x}\right)$

$b - \pi = \tan\frac{1}{x}$

$\cot^{-1}(x) = \tan\frac{1}{x} + \pi$ QED

"FREEDOM" COVER SHEET 95

Name _____

Per _____ Date _____

1) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

2) $\frac{\pi}{2}$

3) $\frac{\pi}{2}\ \frac{3\pi}{2}$

4) $0\ \pi\ \{0, \frac{\pi}{2}, \pi\}$

5) $\frac{\pi}{4}\ \frac{\pi}{2}\ \frac{5\pi}{4}\ \frac{3\pi}{2}$

6) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

7) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

8) ϕ

9) $\frac{\pi}{3}\ \frac{2\pi}{3}\ \frac{4\pi}{3}\ \frac{5\pi}{3}$

10) 0

11) $\frac{3\pi}{2}$

12) $\frac{\pi}{3}\ \frac{\pi}{2}\ \frac{3\pi}{2}\ \frac{5\pi}{3}$

13) $0\ \pi$

14) $0\ \pi$

15) $\frac{2\pi}{3}\ \frac{4\pi}{3}$

16) $0\ \frac{\pi}{3}\ \frac{2\pi}{3}\ \pi$

17) π

18) $0\ \frac{\pi}{3}\ \pi\ \frac{5\pi}{3}$

19) $\frac{\pi}{3}\ \pi\ \frac{5\pi}{3}$

20) $0\ \frac{\pi}{2}\ \pi$

21) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

22) $\frac{\pi}{2}\ \frac{3\pi}{2}$

23) $0\ \frac{3\pi}{2}$

24) $0\ \pi\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

25) $\frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

26) ϕ

27) $\frac{\pi}{4}\ \frac{3\pi}{4}\ \frac{5\pi}{4}\ \frac{7\pi}{4}$

28) $0\ \pi\ \{0, \frac{\pi}{2}, \pi\}$

29) $0\ \pi$

30) $\frac{5\pi}{6}\ \frac{11\pi}{6}\ \{\frac{\pi}{6}, \frac{7\pi}{6}\}$

31) $\frac{7\pi}{6}\ \frac{11\pi}{6}$

32) $\{\frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}\}\ \frac{\pi}{2}\ \frac{2\pi}{3}\ \frac{4\pi}{3}\ \frac{3\pi}{2}$

33) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

34) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

35) $\frac{\pi}{4}\ \frac{5\pi}{4}$

36) $\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}\ 2+\sqrt{3}$

37) $0\ \frac{\pi}{2}$

38) ϕ

39) $\pi\ \frac{5\pi}{3}$

40) $\frac{\pi}{3}\ \pi\ \frac{5\pi}{3}$

41) $0\ \frac{\pi}{4}\ \pi\ \frac{5\pi}{4}$

42) $0\ \pi$

43) $\frac{5\pi}{24}\ \frac{11\pi}{24}\ \frac{29\pi}{24}\ \frac{35\pi}{24}$

44) $0\ \frac{\pi}{6}\ \frac{5\pi}{6}\ \pi$

45) $\frac{\pi}{6}\ \frac{5\pi}{6}$ $\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$

46) $\frac{\pi}{2}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

47) $\frac{\pi}{2}\ \frac{3\pi}{4}\ \frac{3\pi}{2}\ \frac{7\pi}{4}$

48) $\frac{7\pi}{6}\ \frac{11\pi}{6}$

49) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

50) $\frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{3\pi}{2}\ \frac{11\pi}{6}$

51) $0\ \frac{2\pi}{3}\ \frac{4\pi}{3}$

52) $\frac{2\pi}{3}\ \pi\ \frac{4\pi}{3}$

53) $\frac{\pi}{6}\ \frac{5\pi}{6}$

54) $\frac{\pi}{2}$ $\{\frac{\pi}{2}, \frac{3\pi}{2}\}$

55) $\frac{\pi}{4}\ \frac{7\pi}{4}$

56) $\frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}$

57) $\frac{1}{2}$

58) ϕ $\{\frac{\pi}{2}\}$

59) $\frac{\pi}{2}\ \frac{3\pi}{2}$

60) $0\ \frac{3\pi}{8}\ \frac{\pi}{2}\ \frac{2\pi}{8}\ \frac{11\pi}{8}\ \frac{3\pi}{2}\ \frac{15\pi}{8}$

61) $0 \le x < 2\pi$

62) $\frac{7\pi}{6}\ \frac{11\pi}{6}$

63) $\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$ 0

64) $0\ \frac{\pi}{4}\ \frac{\pi}{2}\ \frac{5\pi}{4}\ \frac{3\pi}{2}$

65) $\pi, \frac{7\pi}{6}\ \frac{11\pi}{6}$

66) $\frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

67) $\frac{\pi}{4}\ \frac{3\pi}{4}\ \frac{5\pi}{4}\ \frac{7\pi}{4}$

68) $\frac{25}{5}$

69) $\frac{\pi}{2}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

70) 0

71) $\frac{\pi}{3}\ \frac{5\pi}{3}$

72) $\frac{2\pi}{3}\ \frac{4\pi}{3}$

73) 2 $\{-2\}$

74) $\frac{\pi}{10}\ \frac{\pi}{2}\ \frac{9\pi}{10}\ \frac{13\pi}{10}\ \frac{17\pi}{10}$

75) $\frac{\pi}{12}\ \frac{\pi}{4}\ \frac{5\pi}{12}\ \frac{7\pi}{12}\ \frac{3\pi}{4}\ \frac{11\pi}{12}\ \frac{13\pi}{12}\ \frac{5\pi}{4}\ \frac{17\pi}{12}\ \frac{19\pi}{12}\ \frac{7\pi}{4}\ \frac{23\pi}{12}$

76) $\frac{\sqrt{3}}{6}$

77) $\pm\frac{3\sqrt{10}}{10}$

78) $\frac{\pi}{12}\ \frac{11\pi}{12}\ \frac{13\pi}{12}\ \frac{23\pi}{12}$

79) $\frac{5\sqrt{3}+12}{26}$

80) $0\ \frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}\ \pi\ \frac{7\pi}{6}\ \frac{3\pi}{2}\ \frac{11\pi}{6}$

81) $\pm\sqrt{\frac{34-5\sqrt{34}}{68}}$

82) $0 \le x < 2\pi\ x \neq \pi$

83) $\frac{\pi}{2}\ \frac{3\pi}{2}$

84) $\frac{\pi}{6}\ \frac{\pi}{3}\ \frac{2\pi}{3}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{4\pi}{3}\ \frac{5\pi}{3}\ \frac{11\pi}{6}$

85) $0\ \frac{\pi}{4}\ \frac{\pi}{2}\ \pi\ \frac{5\pi}{4}\ \frac{3\pi}{2}$

86) $0\ \frac{2\pi}{3}\ \frac{4\pi}{3}$

87) $\frac{\pi}{9}\ \frac{2\pi}{9}\ \frac{4\pi}{9}\ \frac{5\pi}{9}\ \frac{7\pi}{9}\ \frac{8\pi}{9}\ \frac{10\pi}{9}\ \frac{11\pi}{9}\ \frac{13\pi}{9}\ \frac{14\pi}{9}\ \frac{16\pi}{9}\ \frac{17\pi}{9}$

88) $\frac{9}{4}, 4$

89) $0 \le x < 2\pi\ x \neq \{\frac{\pi}{2}, \frac{3\pi}{2}\}$

90) $0\ \frac{\pi}{3}\ \frac{2\pi}{3}\ \pi$

91) 16

92) $0 \le x < 2\pi\ x \neq \{\frac{\pi}{2}, \frac{3\pi}{2}\}$

93) $\frac{3\pi}{10}\ \frac{7\pi}{10}\ \frac{3\pi}{2}\ \frac{19\pi}{10}$

4 28 30 32 35

46 54 58 62 73

-0 all +

-10 all ✓

-20 all 0

-30 all −

"FREEDOM" COVER SHEET

10 went to 35 from

Name Pre 95 → 95

Per _____ Date _____

1) Same	2) Same	3) Same	4) Same
5) Same	6) Same	7) 20	8) 26
9) 16	10) 35	11) 24	12) 27
13) 34	14) 18	15) Same	16) 32
17) Same	18) 14	19) 3	20) 7
21) 9	22) 31	23) Same	24) 11
25) 30	26) 8	27) 12	28) 28
29) Same	30) 25	31) 22	32) 21
33) 19	34) 13	35) 10	36) Same
37) 57	38) 52	39) 54	40) 58
41) 53	42) 71	43) Same	44) 51
45) 48	46) 60	47) 56	48) 45
49) Same	50) 42	51) 44	52) 28
53) 41	54) 39	55) Same	56) 80
57) 37	58) 40	59) Same	60) 46
61)	62)	63)	64)
65) Same	66)	67)	68)
69)	70) 40	71) 50	72)
73)	74) New	75) New	76)
77)	78) New	79)	80) New
81)	82)	83)	84)
	86) New	87) New	88)
89)	90) New	91)	92)
93) New			

2.3
66.10
6.11.2.5

3.72

"FREEDOM RINGS"

There are 76 required problems. Total value is 10 assignments of "+." Each error or omission reduces the 10 by 1 level. (Total 10 wrong = 10 checks; total 15 wrong = 5 checks, 5 zeros; etc.) An error in recording your answer on the answer sheet *is* an error. An answer on the answer sheet without the proper work is an error on the answer sheet *and* an error on the work! (i.e. 2 errors). There are 17 "extra" problems. These will forgive up to 6 errors.

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When you have completed this task, you will be **free** of any doubts concerning your ability to use algebra, your knowledge of trigonometric identities, and your prowess with inverse trig functions! You have joined many patriots before you. Paul Revere, American patriot, skilled craftsman, silversmith, engraver, and printer, (before he made cooking pots and pans called *Revereware*) celebrated completion of his homework with a ride warning Massachusetts colonists that British troops were marching toward Lexington and Concord to seize militia supplies and that he had completed **Freedom Rings**. He was immortalized in a Longfellow poem. The facts about his math work had been lost until now.

1) $2\sin^2 x - 1 = -\sin x$
2) $2\cos^2 x = \sin x - 1$
3) $\sin^2 x = 1$
4) $\sin x = \tan x$
5) $\cot^2 x = \cot x$
6) $\cot x \tan 2x = 3$
7) $\sin^2 x = \sin x$
8) $2\sin x = 3$
9) $2\sin^2 x - \sqrt{3}\sin x = 0$
10) $3\tan x - 4 = \tan x - 2$
11) $2\sin^2 x = -\sin x$
12) $\tan x = \cot x$
13) $\sec^2 x = \frac{4}{3}$
14) $2\sin x \cos x = \sin x$
15) $2\cos x + 1 = 0$
16) $\cos x - 2\sin^2 x + 2 = 0$
17) $2\sin^2 x + \cos x = -\cos^2 x$
18) $\cos x \tan x = 3\tan x$
19) $2 + 2\sin x = 4\cos^2 x$
20) $4\cos^2 x - 3 = 0$
21) $\cos^2 x = \frac{1}{4}$
22) $2\sin x + 1 = 0$
23) $\sin x \cos x + \cos x - 1 - \sin x = 0$
24) $2\sin^2 x - \sin x = 3$
25) $\sqrt{3}\tan x = -1$
26) $(3\sec x - 2)(\cos x) = 0$
27) $2\cos^2 x = \cos x$
28) $\sin x + \sec^2 x = 1 + \sin^2 x + \tan^2 x$
29) $\cos^2 x - \sin^2 x = 1$
30) $\sin 2x = \cos x$
31) $\sin^2 \frac{x}{2} + \tan^2 \frac{x}{2} = \frac{3}{2}$
32) $\cos 2x = \sin x$
33) $2\sin^2 x - \cos x = 1$
34) $\sin 2x = 2\sin x$
35) $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$
36) $\arctan(x-2) = \frac{\pi}{3}$

"FREEDOM RINGS"

37) $\cos^{-1} 2x = \frac{\pi}{3} - \cos^{-1} x$

38) $2\cos^2 x + 3\cos x + 1 = 0$

39) $\sin^2 x - 2\sin x + 1 = 0$

40) $\tan^{-1} x + \tan^{-1} 1 = \frac{3\pi}{4}$

41) $2\sin x \sec x = \sec x$

42) $3\tan\frac{x}{2} = \cot\frac{x}{2}$

43) $\sin^2\left(x+\frac{\pi}{6}\right) - \cos^2\left(x+\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$

44) $\cos 2x + 2\sin^2\frac{x}{2} = 1$

45) $\csc^2 x + 2\csc x = 0$

46) $\sec^2 2x + \tan 2x = 1$

47) $2\sin^2 x - 3\sin x + 1 = 0$

48) $2\sin^2 x + 3\sin x - 2 = 0$

49) $\tan 2x \tan x = 1$

50) $2\cos^2 x = 1 + \cos^2 x$

51) $\cos 2x + \sin x - 1 = 0$

52) $\tan\left(\frac{\pi}{4} + 3x\right) = 2\tan 6x$

53) $\sin 2x + \cos 2x - 1 = 0$

54) $-\sqrt{3}\sin x - \cos x = 1$

55) $2\sec x - 2\sqrt{2} = 0$

56) $\sin 2x \tan^2 2x - \tan 2x = \sin 2x$

57) $\sin x + \cos x = 1$

58) $\sin 2x \sin x - \cos 2x \cos x = 1$

59) $2\sin\left(x - \frac{\pi}{6}\right) = \sqrt{3}\sin x$

60) $2\sin^2 x - \sin x - 1 = 0$

61) $\sin^2\left(x+\frac{\pi}{2}\right) + \cos^2\left(x+\frac{\pi}{2}\right) - 1 = 0$

62) $2\cos 2x + \sin 2x \sec x = 0$

63) $3\sec x - \cos x = 2$

64) $2\cos^2 x \sin^2 x - \sin x \cos x = 0$

65) $\sin 2x + \cos x + 2\sin x = -1$

66) $\sin(\pi - 2x) = \cos x$

67) $2\tan x \sin x + 2\sin x = \tan x + 1$

68) $\tan\left(\pi + \arcsin\frac{2}{3}\right) = x$

69) $\sin x = \cos(2x - \pi)$

70) $\sin 2x \sin x + \cos x \cos 2x = 1$

71) $\cos^2\left(x+\frac{\pi}{3}\right) - \sin^2\left(x+\frac{\pi}{3}\right) = \frac{1}{2}$

72) $2 + \sec x = 0$

73) $\tan\left(\tan^{-1} x + \tan^{-1} 1\right) = -3$

74) $\tan\left(\cos^{-1} u\right) = x$

75) $\cos\left(\sin^{-1} u\right) = x$

76) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{2} - \sin^{-1} 3x$

77) $\cos\left(\frac{1}{2}\arctan\frac{3}{4}\right) = x$

78) $\sin\left(\cos^{-1} u\right) = x$

79) $\cos\left(\arcsin\frac{-1}{2} + \cos^{-1}\frac{5}{13}\right) = x$

80) $\sin 2x - \sin 4x = 2\sin x$

81) $\sin\left(\frac{1}{2}\arctan\frac{3}{5}\right) = x$

82) $\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$

83) $2\cos x = 4\cos x \csc x$

84) $\tan 2x = 3\cot 2x$

85) $\cos^3 x - \sin^3 x = (\cos x - \sin x)\left(1 + \frac{1}{2}\sin 2x\right)$

86) $\cot\left(\cos^{-1} x\right) = x$

87) $\sin\left(\sec^{-1} u\right) = x$

88) $\sec^{-1}\frac{5}{3} + \csc^{-1}\frac{5}{4} = 2\tan^{-1}\frac{x}{3}$

89) $\tan^2 x - \sec^2 x = -1$

90) $\tan\left(\csc^{-1} u\right) = x$

91) $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{12}{13} = \cos^{-1}\frac{-x}{65}$

92) $1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$

93) $\cos\left(\csc^{-1} u\right) = x$

HONORS TRIGONOMETRY 95/96 1776

"FREEDOM RINGS" Chapter 4

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3) $\sin^2 x = 1$

4) $\sin x = \tan x$

5) $\cot^2 x = \cot x$

6) $\cot x \tan 2x = 3$

7) $4\cos^2 x - 3 = 0$

8) $(3\sec x - 2)(\cos x) = 0$

9) $\cos^2 x = \frac{1}{4}$

10) $\tan^{-1}(1 + x) + \tan^{-1}(1 - x) = \frac{\pi}{2}$

11) $2\sin^2 x - \sin x = 3$

12) $2\cos^2 x = \cos x$

13) $\sin 2x = 2\sin x$

14) $\cos x \tan x = 3\tan x$

15) $2\cos x + 1 = 0$

16) $2\sin^2 x - \sqrt{3}\sin x = 0$

17) $2\sin^2 x + \cos x = -\cos^2 x$

18) $2\sin x \cos x = \sin x$

19) $2\sin^2 x - \cos x = 1$

20) $\sin^2 x = \sin x$

21) $\cos 2x = \sin x$

22) $\sin^2 \frac{x}{2} + \tan^2 \frac{x}{2} = \frac{3}{2}$

23) $\sin x \cos x + \cos x = 1 + \sin x$

24) $2\sin^2 x = -\sin x$

25) $\sin 2x = \cos x$

26) $2\sin x = 3$

27) $\tan x = \cot x$

28) $\sin x + \sec^2 x = 1 + \sin^2 x + \tan^2 x$

29) $\cos^2 x - \sin^2 x = 1$

30) $\sqrt{3}\tan x = -1$

31) $2\sin x + 1 = 0$

32) $\cos x - 2\sin^2 x + 2 = 0$

33) $2 + 2\sin x = 4\cos^2 x$

34) $\sec^2 x = \frac{4}{3}$

35) $3\tan x - 4 = \tan x - 2$

36) $\arctan(x - 2) = \frac{\pi}{3}$

37) $\sin x + \cos x = 1$

38) $\tan\left(\frac{\pi}{4} + 3x\right) = 2\tan 6x$

39) $-\sqrt{3}\sin x - \cos x = 1$

40) $\sin 2x \sin x - \cos 2x \cos x = 1$

41) $\sin 2x + \cos 2x = 1$

42) $2\cos^2 x = 1 + \cos^2 x$

43) $\sin^2\left(x + \frac{\pi}{6}\right) - \cos^2\left(x + \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$

44) $\cos 2x + \sin x = 1$

45) $2\sin^2 x + 3\sin x - 2 = 0$

46) $2\sin^2 x - \sin x - 1 = 0$

47) $2\cot^2 x + 2\cot x = 0$

48) $\csc^2 x + 2\csc x = 0$

49) $\tan 2x \tan x = 1$

50) $\cos^2\left(x + \frac{\pi}{3}\right) - \sin^2\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$

51) $\cos 2x + 2\sin^2 \frac{x}{2} = 1$

52) $2\cos^2 x + 3\cos x = -1$

53) $2\sin x \sec x = \sec x$

54) $\sin^2 x - 2\sin x + 1 = 0$

55) $2\sec x - 2\sqrt{2} = 0$

56) $2\sin^2 x - 3\sin x + 1 = 0$

57) $\cos^{-1} 2x = \frac{\pi}{3} - \cos^{-1} x$

58) $\tan^{-1} x + \tan^{-1} 1 = \frac{3\pi}{4}$

59) $2\sin\left(x - \frac{\pi}{6}\right) = \sqrt{3}\sin x$

60) $\sec^2 2x + \tan 2x = 1$

61) $\sin^2\left(x + \frac{\pi}{2}\right) + \cos^2\left(x + \frac{\pi}{2}\right) - 1 = 0$

62) $2\cos 2x + \sin 2x \sec x = 0$

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70) $\sin 2x \sin x + \cos x \cos 2x = 1$

71) $3\tan \frac{x}{2} = \cot \frac{x}{2}$

72) $2 + \sec x = 0$

73) $\tan\left(\tan^{-1} x + \tan^{-1} 1\right) = -3$

74) $\sin 2x \cos 3x + \cos 2x \sin 3x = 1$

75) $\tan^2 3x = 1$

76) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{2} - \sin^{-1} 3x$

77) $\cos\left(\frac{1}{2}\arctan \frac{3}{4}\right) = x$

78) $2\cos 2x = \sqrt{3}$

79) $\cos\left(\arcsin \frac{-1}{2} + \cos^{-1} \frac{5}{13}\right) = x$

80) $\sin 2x \tan^2 2x - \tan 2x = \sin 2x$

81) $\sin\left(\frac{1}{2}\arctan \frac{3}{5}\right) = x$

82) $\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$

83) $2\cos x = 4\cos x \csc x$

84) $\tan 2x = 3\cot 2x$

85) $\cos^3 x - \sin^3 x = (\cos x - \sin x)\left(1 + \frac{1}{2}\sin 2x\right)$

86) $\cos x = \cos 2x$

87) $\tan^2 3x = 3$

88) $\sec^{-1} \frac{5}{3} + \csc^{-1} \frac{5}{4} = 2\tan^{-1} \frac{x}{3}$

89) $\tan^2 x - \sec^2 x = -1$

90) $\sqrt{3}\tan x = 2\sin x \tan x$

91) $\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{12}{13} = \cos^{-1} \frac{-x}{65}$

92) $1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$

93) $\sin^3 5x = -1$

"FREEDOM" 95

COVER SHEET

Name _____

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1) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

2) $\frac{\pi}{2}$

3) $\frac{\pi}{2}\ \frac{3\pi}{2}$

4) $0\ \pi\ \{0, \frac{\pi}{2}, \pi$

5) $\frac{\pi}{4}\ \frac{\pi}{2}\ \frac{5\pi}{4}\ \frac{3\pi}{2}$

6) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

7) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

8) ϕ

9) $\frac{\pi}{3}\ \frac{2\pi}{3}\ \frac{4\pi}{3}\ \frac{5\pi}{3}$

10) 0

11) $\frac{3\pi}{2}$

12) $\frac{\pi}{3}\ \frac{\pi}{2}\ \frac{3\pi}{2}\ \frac{5\pi}{3}$

13) $0\ \pi$

14) $0\ \pi$

15) $\frac{2\pi}{3}\ \frac{4\pi}{3}$

16) $0\ \frac{\pi}{3}\ \frac{2\pi}{3}\ \pi$

17) π

18) $0\ \frac{\pi}{3}\ \pi\ \frac{5\pi}{3}$

19) $\frac{\pi}{3}\ \pi\ \frac{5\pi}{3}$

20) $0\ \frac{\pi}{2}\ \pi$

21) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

22) $\frac{\pi}{2}\ \frac{3\pi}{2}$

23) $0\ \frac{3\pi}{2}$

24) $0\ \pi\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

25) $\frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

26) ϕ

27) $\frac{\pi}{4}\ \frac{3\pi}{4}\ \frac{5\pi}{4}\ \frac{7\pi}{4}$

28) $0\ \pi\ \{0, \frac{\pi}{2}$

29) $0\ \pi$

30) $\frac{5\pi}{6}\ \frac{11\pi}{6}$

31) $\{\frac{\pi}{6}, \frac{7\pi}{6}\}$

32) $\frac{7\pi}{6}\ \frac{11\pi}{6}$

$\{\frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}\}$ $\frac{\pi}{2}\ \frac{2\pi}{3}\ \frac{4\pi}{3}\ \frac{3\pi}{2}$

33) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

34) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

35) $\frac{\pi}{4}\ \frac{5\pi}{4}$

36) $2+\sqrt{3}$

$\frac{\pi}{4}\ \frac{3\pi}{4}\ \frac{5\pi}{4}\ \frac{7\pi}{4}$

37) $0\ \frac{\pi}{2}$

38) ϕ

39) $\pi\ \frac{5\pi}{3}$

40) $\frac{\pi}{3}\ \pi\ \frac{5\pi}{3}$

41) $0\ \frac{\pi}{4}\ \pi\ \frac{5\pi}{4}$

42) $0\ \pi$

43) $\frac{5\pi}{24}\ \frac{13\pi}{24}\ \frac{25\pi}{24}\ \frac{35\pi}{24}$

44) $0\ \frac{\pi}{6}\ \frac{5\pi}{6}\ \pi$

$\frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

46) $\frac{\pi}{6}\ \frac{5\pi}{6}$

47) $\frac{\pi}{2}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

48) $\frac{\pi}{2}\ \frac{3\pi}{4}\ \frac{3\pi}{2}\ \frac{7\pi}{4}$

49) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

50) $\frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{3\pi}{2}\ \frac{11\pi}{6}$

51) $0\ \frac{2\pi}{3}\ \frac{4\pi}{3}$

52) $\frac{2\pi}{3}\ \pi\ \frac{4\pi}{3}$

53) $\frac{\pi}{6}\ \frac{5\pi}{6}$

54) $\frac{\pi}{2}$

55) $\{\frac{\pi}{2}, \frac{3\pi}{2}\}$

56) $\frac{\pi}{4}\ \frac{7\pi}{4}$

57) $\frac{1}{2}$

58) ϕ

59) $\{\frac{\pi}{2}\}$

60) $\frac{\pi}{2}\ \frac{3\pi}{2}$

$0\ \frac{3\pi}{8}\ \frac{\pi}{2}\ \frac{7\pi}{8}\ \frac{11\pi}{8}\ \frac{3\pi}{2}\ \frac{15}{8}$

61) $0 \leq x < 2\pi$

62) $\frac{7\pi}{6}\ \frac{11\pi}{6}$

63) $\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

64) 0

$0\ \frac{\pi}{4}\ \frac{\pi}{2}\ \frac{5\pi}{4}\ \frac{3\pi}{2}$

65) $\pi, \frac{7\pi}{6}\ \frac{11\pi}{6}$

66) $\frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$

67) $\frac{\pi}{6}\ \frac{3\pi}{4}\ \frac{5\pi}{6}\ \frac{7\pi}{4}$

68) $\frac{2\sqrt{5}}{5}$

69) $\frac{\pi}{2}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$

70) 0

71) $\frac{\pi}{3}\ \frac{5\pi}{3}$

72) $\frac{2\pi}{3}\ \frac{4\pi}{3}$

73) $2\ \{-2\}$

74) $\frac{\pi}{10}\ \frac{\pi}{2}\ \frac{9\pi}{10}\ \frac{13\pi}{10}\ \frac{17\pi}{10}$

75) $\frac{\pi}{12}\ \frac{\pi}{4}\ \frac{5\pi}{12}\ \frac{7\pi}{12}\ \frac{3\pi}{4}\ \frac{11\pi}{12}$

$\frac{13\pi}{12}\ \frac{5\pi}{4}\ \frac{17\pi}{12}\ \frac{19\pi}{12}\ \frac{13\pi}{4}\ \frac{23\pi}{12}$

76) $\frac{\sqrt{3}}{6}$

77) $\pm\frac{3\sqrt{10}}{10}$

78) $\frac{\pi}{12}\ \frac{11\pi}{12}\ \frac{13\pi}{12}\ \frac{23\pi}{12}$

79) $\frac{5\sqrt{3}+12}{26}$

80) $0\ \frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}\ \pi\ \frac{7\pi}{6}\ \frac{3\pi}{2}$

$\frac{11\pi}{6}$

81) $\pm\sqrt{\frac{34-5\sqrt{34}}{68}}$

82) $0 \leq x < 2\pi\ x \neq \pi$

83) $\frac{\pi}{2}\ \frac{3\pi}{2}$

84) $\frac{\pi}{6}\ \frac{\pi}{3}\ \frac{2\pi}{3}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{4\pi}{3}\ \frac{5\pi}{3}\ \frac{11}{6}$

$\frac{\pi}{4}\ \frac{2\pi}{4}\ \frac{3\pi}{4}\ \frac{5\pi}{4}\ \frac{6\pi}{4}\ \frac{7\pi}{4}\ \frac{8\pi}{4}$

$\frac{\pi}{9}\ \frac{2\pi}{9}\ \frac{4\pi}{9}\ \frac{5\pi}{9}\ \frac{7\pi}{9}\ \frac{8\pi}{9}\ \frac{10\pi}{9}$

$\frac{11\pi}{9}\ \frac{13\pi}{9}\ \frac{14\pi}{9}\ \frac{16\pi}{9}\ \frac{17\pi}{9}$

85) $0\ \frac{\pi}{4}\ \frac{\pi}{2}\ \pi\ \frac{5\pi}{4}\ \frac{3\pi}{2}$

86) $0\ \frac{2\pi}{3}\ \frac{4\pi}{3}$

87) $\frac{\pi}{9}\ \frac{2\pi}{9}\ \frac{4\pi}{9}\ \frac{5\pi}{9}\ \frac{7\pi}{9}\ \frac{8\pi}{9}\ \frac{10\pi}{9}$

$\frac{11\pi}{9}\ \frac{13\pi}{9}\ \frac{14\pi}{9}\ \frac{16\pi}{9}\ \frac{17\pi}{9}$

88) $\frac{9}{4}, 4$

89) $0 \leq x < 2\pi\ x \neq \{\frac{\pi}{2}, \frac{3\pi}{2}\}$

90) $0\ \frac{\pi}{3}\ \frac{2\pi}{3}\ \pi$

91) 16

92) $0 \leq x < 2\pi\ x \neq \{\frac{\pi}{2}, \frac{3\pi}{2}$

93) $\frac{3\pi}{10}\ \frac{7\pi}{10}\ \frac{3\pi}{2}\ \frac{19\pi}{10}$

4 28 30 32 35

46 54 58 62 73

"FREEDOM" COVER SHEET

Name Charlie Heston

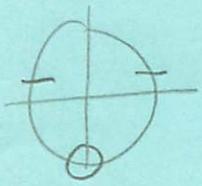
Per _____ Date _____

1) $\frac{\pi}{2}\ \frac{5\pi}{2}\ \frac{3\pi}{2}$
2) $\frac{\pi}{2}$
3) $\frac{\pi}{2}\ \frac{3\pi}{2}$
4) $0\ \frac{\pi}{2}\ \pi$
5) $\frac{\pi}{4}\ \frac{\pi}{2}\ \frac{5\pi}{4}\ \frac{3\pi}{2}$
6) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
7) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
8) ϕ
9) $\frac{\pi}{3}\ \frac{2\pi}{3}\ \frac{4\pi}{3}\ \frac{5\pi}{3}$
10) 0
11) $\frac{3\pi}{2}$
12) $\frac{\pi}{3}\ \frac{\pi}{2}\ \frac{3\pi}{2}\ \frac{5\pi}{3}$
13) $0\ \pi$
14) $0\ \pi$
15) $\frac{2\pi}{3}\ \frac{4\pi}{3}$
16) $0\ \frac{\pi}{3}\ \frac{2\pi}{3}\ \pi$
17) π
18) $0\ \frac{\pi}{3}\ \pi\ \frac{5\pi}{3}$
19) $\frac{\pi}{3}\ \pi\ \frac{5\pi}{3}$
20) $0\ \frac{\pi}{2}\ \pi$
21) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$
22) $\frac{\pi}{2}\ \frac{3\pi}{2}$
23) $0\ \frac{3\pi}{2}$
24) $0\ \pi\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
25) $\frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$
26) ϕ
27) $\frac{\pi}{4}\ \frac{3\pi}{4}\ \frac{5\pi}{4}\ \frac{7\pi}{4}$
28) $0\ \frac{\pi}{2}\ \pi$
29) $0\ \pi$
30) $\frac{\pi}{6}\ \frac{7\pi}{6}$
31) $\frac{7\pi}{6}\ \frac{11\pi}{6}$
32) $\frac{\pi}{3}\ \frac{\pi}{2}\ \frac{3\pi}{2}\ \frac{5\pi}{3}$
33) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$
34) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
35) $\frac{\pi}{4}\ \frac{3\pi}{4}\ \frac{5\pi}{4}\ \frac{7\pi}{4}$
36) $2+\sqrt{3}$
37) $0\ \frac{\pi}{2}$
38) ϕ
39) $\pi\ \frac{5\pi}{3}$
40) $\frac{\pi}{3}\ \pi\ \frac{5\pi}{3}$
41) $0\ \frac{\pi}{4}\ \pi\ \frac{5\pi}{4}$
42) $0\ \pi$
43) $\frac{5\pi}{24}\ \frac{11\pi}{24}\ \frac{29\pi}{24}\ \frac{35\pi}{24}$
44) $0\ \frac{\pi}{6}\ \frac{5\pi}{6}\ \pi$
46) $\frac{\pi}{6}\ \frac{5\pi}{6}$
47) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$
48) $\frac{\pi}{2}\ \frac{3\pi}{4}\ \frac{3\pi}{2}\ \frac{7\pi}{4}$
49) $\frac{\pi}{6}\ \frac{5\pi}{6}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
50) $\frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{3\pi}{2}\ \frac{11\pi}{6}$
51) $0\ \frac{2\pi}{3}\ \frac{4\pi}{3}$
52) $\frac{2\pi}{3}\ \pi\ \frac{4\pi}{3}$
53) $\frac{\pi}{6}\ \frac{5\pi}{6}$
54) $\frac{\pi}{2}\ \frac{3\pi}{2}$
55) $\frac{\pi}{4}\ \frac{7\pi}{4}$
56) $\frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}$
57) $\frac{1}{2}$
58) $\frac{\pi}{2}$
59) $\frac{\pi}{2}\ \frac{3\pi}{2}$
60) $\frac{15\pi}{8}\ \frac{3\pi}{8}\ \frac{\pi}{2}\ \frac{7\pi}{8}\ \frac{11\pi}{8}\ \frac{3\pi}{2}$
61) $0 \le x < 2\pi$
62) $\frac{\pi}{2}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
63) 0
64) $0\ \frac{\pi}{4}\ \frac{\pi}{2}\ \frac{5\pi}{4}\ \frac{3\pi}{2}$
65) $\pi\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
66) $\frac{\pi}{6}\ \frac{\pi}{2}\ \frac{5\pi}{6}\ \frac{3\pi}{2}$
67) $\frac{\pi}{6}\ \frac{3\pi}{4}\ \frac{5\pi}{6}\ \frac{7\pi}{4}$
68) $\frac{2\sqrt{5}}{5}$
69) $\frac{\pi}{2}\ \frac{7\pi}{6}\ \frac{11\pi}{6}$
70) 0
71) $\frac{\pi}{3}\ \frac{5\pi}{3}$
72) $\frac{2\pi}{3}\ \frac{4\pi}{3}$
73) -2
74) $\frac{\pi}{10}\ \frac{\pi}{2}\ \frac{9\pi}{10}\ \frac{13\pi}{10}\ \frac{17\pi}{10}$
75) $\frac{\pi}{12}\ \frac{\pi}{4}\ \frac{9\pi}{12}\ \frac{13\pi}{12}\ \frac{3\pi}{4}\ \frac{17\pi}{12}\ \frac{5\pi}{4}\ \frac{21\pi}{12}\ \frac{7\pi}{4}\ \frac{23\pi}{12}$
76) $\frac{\sqrt{3}}{6}$
77) _____
78) _____
79) _____
80) _____
81) _____
82) _____
83) _____
84) _____
86) _____
87) _____
88) _____
89) _____
90) _____
91) _____
92) _____
93) _____

① $2\sin^2 x - 1 = -\sin x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$



$x \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

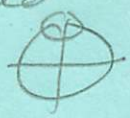
② $2\cos^2 x = \sin x - 1$

$2 - 2\sin^2 x = \sin x - 1$

$2\sin^2 x + \sin x - 3 = 0$

$(2\sin x + 3)(\sin x - 1) = 0$

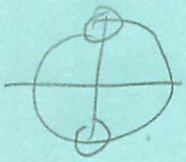
Reject



$x = \frac{\pi}{2}$

③ $\sin^2 x = 1$

$\sin x = \pm 1$

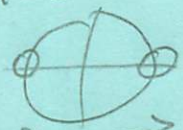


$x \in \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

④ $\sin x = \tan x$

$\sin x\left(\frac{1}{\cos x} - 1\right) = 0$

$\sin x = 0$ $\quad \cos x = 1$

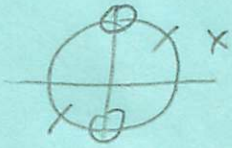


$x \in \{0, \pi\}$

WRONG ANS $0, \frac{\pi}{2}, \pi$

⑤ $\cot^2 x = \cot x$

$\cot x(\cot x - 1) = 0$



$x \in \left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}\right\}$

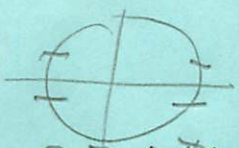
⑥ $\cot x \tan 2x = 3$

$\frac{\cot x \cdot 2\tan x}{1 - \tan^2 x} = 3$

$2 = 3 - 3\tan^2 x$

$3\tan^2 x = 1$

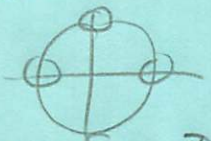
$\tan x = \pm\frac{1}{\sqrt{3}} > \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$



$x \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

⑦ $\sin^2 x = \sin x$

$\sin x(\sin x - 1) = 0$



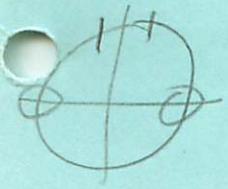
$x \in \left\{0, \frac{\pi}{2}, \pi\right\}$

⑧ $2\sin x = 3$

$x \in \{\}$ Reject $R_{\sin}: |\sin x| \le 1$

⑨ $2\sin^2 x - \sqrt{3}\sin x = 0$

$\sin x(2\sin x - \sqrt{3}) = 0$

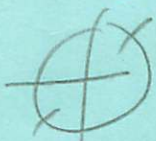


$x \in \left\{0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$

⑩ $3\tan x - 4 > \tan x - 2$

$2\tan x = 2$

$\tan x = 1$



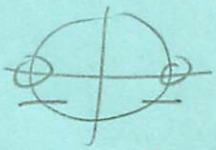
$x \in \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

WRONG answer

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

⑪ $2\sin^2 x = -\sin x$

$\sin x(2\sin x + 1) = 0$



$x \in \left\{0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

(12) $\tan x = \cot x$

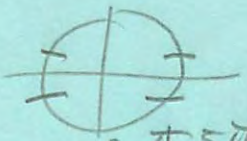
$\tan^2 x = 1$

$\tan x = \pm 1$

$x \in \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$

(13) $\sec^2 x = \frac{4}{3}$

$\sec x = \pm \frac{2}{\sqrt{3}}$



$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

(14) $2\sin x \cos x = \sin x$

$\sin x(2\cos x - 1) = 0$

$x \in \{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\}$

(15) $2\cos x + 1 = 0$

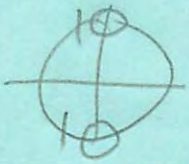
$\cos x = -\frac{1}{2}$

$x \in \{\frac{2\pi}{3}, \frac{4\pi}{3}\}$

(16) $\cos x - 2\sin^2 x + 2 = 0$

$\cos x - 2 + 2\cos^2 x + 2 = 0$

$\cos x(1 + 2\cos x) = 0$



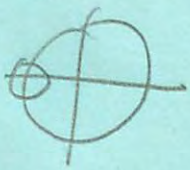
$x \in \{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\}$

(17) $2\sin^2 x + \cos x = -\cos^2 x$

$2 - 2\cos^2 x + \cos x + \cos^2 x = 0$

$\cos^2 x - \cos x - 2 = 0$

$(\cos x + 1)(\cos x + 2) = 0$ (rejected)



$x = \pi$

(18) $\cos x \tan x = 3\tan x$

$\tan x(\cos x - 3) = 0$ (rejected)

$x \in \{0, \pi\}$

wrong ans $\frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$

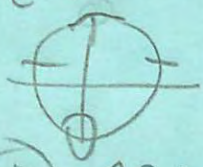
(19) $2 + 2\sin x = 4\cos^2 x$

$= 4 - 4\sin^2 x$

$4\sin^2 x + 2\sin x - 2 = 0$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$



$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$

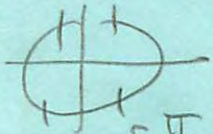
(20) $4\cos^2 x - 3 = 0$

$\cos x = \frac{\pm\sqrt{3}}{2}$

$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

(21) $\cos^2 x = \frac{1}{4}$

$\cos x = \pm\frac{1}{2}$



$x \in \{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\}$

(22) $2\sin x + 1 = 0$

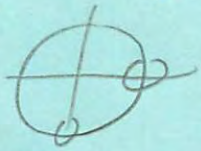
$\sin x = -\frac{1}{2}$

$x \in \{\frac{7\pi}{6}, \frac{11\pi}{6}\}$

(23) $\sin x \cos x + \cos x + 1 - \sin x = 0$

$\cos x(\sin x + 1) - (\sin x + 1) = 0$

$(\cos x - 1)(\sin x + 1) = 0$



$x \in \{0, \frac{3\pi}{2}\}$

(24) $2\sin^2 x - \sin x = 3$

$(2\sin x - 3)(\sin x + 1)$ (rejected)

$x = \frac{3\pi}{2}$.

(25) $\sqrt{3}\tan x = -1$

$\tan x = -\frac{1}{\sqrt{3}}$ $-\frac{1}{\frac{\sqrt{3}}{2}}$

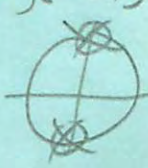
$x \in \{\frac{5\pi}{6}, \frac{11\pi}{6}\}$

wrong answer

$\frac{\pi}{6}, \frac{7\pi}{6}$

(26) $(3\sec x - 2)(\cos x) = 0$ (rejected)

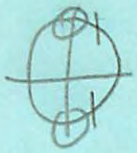
$|\sec x| \geq 1$



Reject ϕ

(27) $2\cos^2 x = \cos x$

$\cos x(2\cos x - 1) = 0$



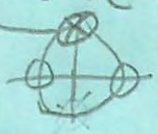
$x \in \left\{\frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}\right\}$

(28) $\sin x + \sec^2 x = 1 + \tan^2 x + \tan x$

$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2$

$\sin x(\sin x - 1) = 0$

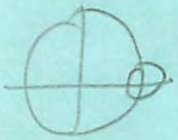


$x \in \{0, \pi\}$

wrong answers $0, \frac{\pi}{2}, \pi$

(29) $\cos^2 x - \sin^2 x = 1$

$\cos 2x = 1$



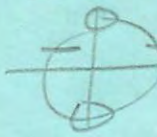
$2x \in \{0, 2\pi\}$

$x \in \{0, \pi\}$

(30) $\sin 2x = \cos x$

$2\sin x \cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$



$x \in \left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

(31) $\sin^2 \frac{x}{2} + \tan^2 \frac{x}{2} = \frac{3}{2}$

$1 - \cos^2 \frac{x}{2} + \tan^2 \frac{x}{2} = \frac{3}{2}$

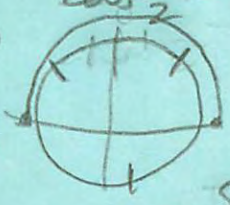
$\sec^2 \frac{x}{2} - \cos^2 \frac{x}{2} = \frac{3}{2}$

$2 - 2\cos^4 \frac{x}{2} = 3\cos^2 \frac{x}{2}$

$2\cos^4 \frac{x}{2} + 3\cos^2 \frac{x}{2} - 2 = 0$

$\left(2\cos^2 \frac{x}{2} - 1\right)\left(\cos^2 \frac{x}{2} + 2\right)$

$\cos \frac{x}{2} = \pm \frac{1}{\sqrt{2}}$ $\quad \cos^2 \frac{x}{2} = -2$ Reject



$x \in \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

$\frac{x}{2} = \left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

(32) $\cos 2x = \sin x$

$\cos^2 x - \sin^2 x = \sin x$

$1 - 2\sin^2 x = \sin x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

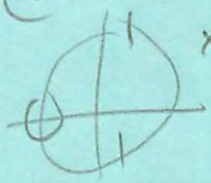
$x \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

(33) $2\sin^2 x - \cos x = 1$

$2 - 2\cos^2 x - \cos x = 1$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

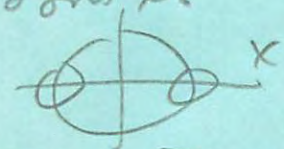


$x \in \left\{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$

(34) $\sin 2x = 2\sin x$

$2\sin x \cos x - 2\sin x = 0$

$2\sin x(\cos x - 1) = 0$



$x \in \{0, \pi\}$

(35) $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$

$\cos(\quad) = \cos \frac{\pi}{2}$

$\cos(\tan^{-1}(1+x))\cos(\tan^{-1}1 + x) - \sin(\tan^{-1}(1+x))\sin(\tan^{-1}1+x)$

$\frac{1}{\sqrt{1+(1+x)^2}} \cdot \frac{1}{\sqrt{1-(1+x)^2}} - \frac{1+x}{\sqrt{1+(1+x)^2}} \cdot \frac{1-x}{\sqrt{1+(1-x)^2}} = 0$

$1 - (1 - x^2) = 0$

$1 - 1 + x^2 = 0$

$x^2 = 0$

$x = 0$

(36) $\arccos(x-2) > \frac{\pi}{3}$

$\tan(\tan^{-1}(x-2)) = \tan\frac{\pi}{3}$

$x - 2 = \sqrt{3}$

$x = 2 + \sqrt{3}$

(37) Switched to 57

$\cos^{-1} 2x = \frac{\pi}{3} - \cos^{-1} x$

$\cos(\cos^{-1} 2x) = \cos\left(\frac{\pi}{3} - \cos^{-1} x\right)$

$2x = \cos\frac{\pi}{3}\cos\cos^{-1} x + \sin\frac{\pi}{3}\sin(\cos^{-1} x)$

$2x = \frac{1}{2}x + \frac{\sqrt{3}}{2}\sqrt{1-x^2}$

$3x = \sqrt{3}\sqrt{1-x^2}$

$9x^2 = 3 - 3x^2$

$12x^2 = 3$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$

$\boxed{x = \frac{1}{2}}$

ck $\frac{1}{2}$

$\cos^{-1} 1 = \frac{\pi}{3} - \cos^{-1}\left(\frac{1}{2}\right)$

$0 = \frac{\pi}{3} - \frac{\pi}{3}$

$0 = 0$ OK

ck $\frac{1}{2}$

$\cos^{-1}(-1) = \frac{\pi}{3} - \cos^{-1}\left(\frac{-1}{2}\right)$

$\pi = \frac{\pi}{3} - \frac{2\pi}{3}$

$\pi = -\frac{\pi}{3}$ NO

(38) $2\cos^2 x + 3\cos x + 1 = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$x = \left\{\frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$

(39) $\sin^2 x - 2\sin x + 1 = 0$

$(\sin x - 1)^2 = 0$

$x = \frac{\pi}{2}$

wrong answer $x \in \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

(40) $\tan^{-1} x + \tan^{-1} 1 = \frac{3\pi}{4}$

$\tan^{-1} x + \frac{\pi}{4} = \frac{3\pi}{4}$

$\tan^{-1} x = \frac{\pi}{2}$

Reject $\tan\frac{\pi}{2}$ DNE.

wrong ans $\frac{\pi}{2}$

(41) $2\sin x \sec x = \sec x$

$\sec x(2\sin x - 1) = 0$

Reject $\sin x = \frac{1}{2}$

$x \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

(42) $3\tan\frac{x}{2} = \cot\frac{x}{2}$

$\tan^2\frac{x}{2} = \frac{1}{3}$

$\tan\frac{x}{2} = \pm\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$

$\frac{x}{2} \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

$x \in \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$.

(43) $\sin^2\left(x + \frac{\pi}{6}\right) - \cos^2\left(x + \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$

$\cos 2\left(x + \frac{\pi}{6}\right) = -\frac{\sqrt{2}}{2}$

start

$0 \le x < 2\pi$

$\frac{\pi}{6} \le x + \frac{\pi}{6} < 2\pi + \frac{\pi}{6}$

$\frac{\pi}{3} \le 2\left(x + \frac{\pi}{6}\right) < 2\left(2\pi + \frac{\pi}{6}\right)$

$2\left(x + \frac{\pi}{6}\right) \in \left\{\frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}\right\}$

$x + \frac{\pi}{6} \in \left\{\frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}\right\}$

$\frac{4}{24}$

$x + \frac{4\pi}{24} \in \left\{\frac{9\pi}{24}, \frac{15\pi}{24}, \frac{23\pi}{24}, \frac{39\pi}{24}\right\}$

$x \in \left\{\frac{5\pi}{24}, \frac{11\pi}{24}, \frac{29\pi}{24}, \frac{35\pi}{24}\right\}$

(44) $\cos 2x + 2\sin^2\frac{x}{2} = 1$

$\cos^2 x - \sin^2 x + 2\sin^2\frac{x}{2} = 1$

$-(1-\cos^2 x) - \sin^2 x +$

(51) $\cos 2x + 2\sin^2\frac{x}{2} = 1$

$\cos^2 x - \sin^2 x + 2\sin^2\frac{x}{2} = 1$

$\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)^2 - \left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2 + 2\sin^2\frac{x}{2} = 1$

$(1 - 2\sin^2\frac{x}{2})$

$1 - 4\sin^2\frac{x}{2} + 4\sin^4\frac{x}{2} - 4\sin^2\frac{x}{2}\cos^2\frac{x}{2} + 2\sin^2\frac{x}{2} = 1$

$-2\sin^2\frac{x}{2} + \sin^4\frac{x}{2} - 4\sin^2\frac{x}{2}\cos^2\frac{x}{2} = 0$

$\sin^2\frac{x}{2}\left(-2 + \sin^2\frac{x}{2} - 4\cos^2\frac{x}{2}\right) = 0$

$\sin^2\frac{x}{2}\left(-2 + 1 - \cos^2\frac{x}{2} - 4\cos^2\frac{x}{2}\right) = 0$

$-\sin^2\frac{x}{2}\left(5\cos^2\frac{x}{2} + 1\right) = 0$

(51) $\cos 2x + 2\sin^2\frac{x}{2} = 1$

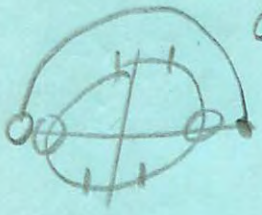
44 Lower RT corner of this Page.

$\cos^2 x - \sin^2 x + 2\sin^2\frac{x}{2} = 1$

$-\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2 + 2\sin^2\frac{x}{2} = \underbrace{1-\cos^2 x}_{\sin^2 x}$

$-2\left(4\sin^2\frac{x}{2}\cos^2\frac{x}{2}\right) + 2\sin^2\frac{x}{2} = 0$

$\sin^2\frac{x}{2}\left(-4\cos^2\frac{x}{2} + 1\right) = 0$



$\cos\frac{x}{2} = \pm\frac{1}{2}$

$\frac{x}{2} \in \left\{0, \frac{\pi}{3}, \frac{2\pi}{3}\right\}$

$x = \left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$.

(47) $2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$x \in \left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}\right\}$

(48) $2\sin^2 x + 3\sin x - 2 = 0$

$(2\sin x - 1)(\sin x + 2) = 0$

Reject

$x \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$.

(50) $2\cos^2 x = 1 + \cos^2 x$

$\cos^2 x = 1$

$\cos x = \pm 1$

$x \in \{0, \pi\}$.

(51) upper Left corner of this page

(45) $\csc^2 x + 2\csc x = 0$

$\csc x(\csc x + 2) = 0$

Reject $\sin x = -\frac{1}{2}$

$x \in \left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

(46) $\sec^2 2x + \tan 2x = 1$

$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2$

$\tan^2 2x + 1 + \tan 2x = 1$

$\tan 2x(\tan 2x + 1) = 0$

$2x \in \left\{0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi, \frac{11\pi}{4}, 3\pi, \frac{15\pi}{4}\right\}$

$x \in \left\{0, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{7\pi}{8}, \pi, \frac{11\pi}{8}, \frac{3\pi}{2}, \frac{15\pi}{8}\right\}$

(49) $\tan 2x \tan x = 1$

$\frac{2\tan^2 x}{1 - \tan^2 y} = 1$

$2\tan^2 y = 1 - \tan^2 y$

$3\tan^2 x = 1$

$\tan x = \pm\frac{1}{\sqrt{3}}$ $\frac{1}{2}$ $\frac{\sqrt{3}}{2}$

$x \in \left\{\frac{\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

(44) $\cos 2x + \sin x - 1 = 0$

$\cos^2 x - \sin^2 x + \sin x - 1 = 0$

$1 - \sin^2 x - \sin^2 x + \sin x - 1 = 0$

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$x \in \left\{0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi\right\}$.

(52) $\tan\left(\frac{\pi}{4}+3x\right)=2\tan 6x$

$$\frac{\tan\frac{\pi}{4}+\tan 3x}{1-\tan\frac{\pi}{4}\tan 3x}=\frac{2\cdot 2\tan 3x}{1-\tan^2 3x}$$

$$\frac{1+\tan 3x}{1-\tan 3x}=\frac{4\tan 3x}{(1-\tan 3x)(1+\tan 3x)}$$

$1+2\tan 3x+\tan^2 3x=4\tan 3x$

$\tan^2 3x-2\tan 3x+1=0$

$(\tan 3x-1)^2=0$

repeat ϕ

(53) $\sin 2x+\cos 2x-1=0$

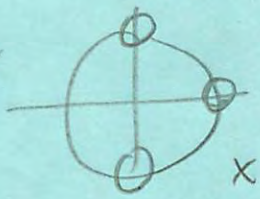
$\sin 2x=1-\cos 2x$

$\sin^2 2x=1-2\cos 2x+\cos^2 2x$

$1-\cos^2 2x=$

$2\cos^2 2x-2\cos 2x=0$

$2\cos 2x(\cos 2x-1)=0$



$2x\in\left\{0,\frac{\pi}{2},\frac{3\pi}{2},\frac{2\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2}\right\}$

$x\in\left\{0,\frac{\pi}{4},\frac{3\pi}{4},\pi,\frac{5\pi}{4},\frac{7\pi}{4}\right\}$

may be extraneous roots. Requires chk of all roots.

0 $\sin 0+\cos 0-1=0$ OK
$0+1-1=0$

$\frac{\pi}{4}$ $\sin\frac{\pi}{2}+\cos\frac{\pi}{2}-1=0$ OK
$1+0-1=0$

$\frac{3\pi}{4}$ $\sin\frac{3\pi}{2}+\cos\frac{3\pi}{2}-1=0$ NO!
$-1+0-1=0$

π $\sin 2\pi+\cos 2\pi-1=0$ OK
$0+1-1=0$

$\frac{5\pi}{4}$ $\sin\frac{5\pi}{2}+\cos\frac{5\pi}{2}-1=0$ OK
$1+0-1=0$

$\frac{7\pi}{4}$ $\sin\frac{7\pi}{2}+\cos\frac{7\pi}{2}-1=0$ NO
$-1+0-1=0$

(54) $-\sqrt{3}\sin x-\cos x=1$

$-\sqrt{3}\sin x=1+\cos x$

$3\sin^2 x=1+2\cos x+\cos^2 x$

$3-3\cos^2 x=$

$4\cos^2 x+2\cos x-2=0$

$2\cos^2 x+\cos x-1=0$

$(2\cos x-1)(\cos x+1)=0$

$x\in\left\{\frac{\pi}{3},\pi,\frac{5\pi}{3}\right\}$

$x\in\left\{\pi,\frac{5\pi}{3}\right\}$

chk for (54) $\frac{\pi}{3}$ $-\sqrt{3}\sin\frac{\pi}{3}-\cos\frac{\pi}{3}=1$

$-\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{2}\right)=1$

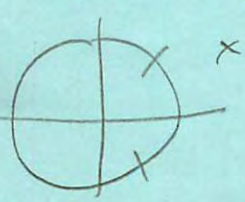
$-\frac{3}{2}-\frac{1}{2}=1$ NO

others check.

(55) $2\sec x-2\sqrt{2}=0$

$\sec x=\sqrt{2}$

$\cos=\frac{\sqrt{2}}{2}$



$x\in\left\{\frac{\pi}{4},\frac{7\pi}{4}\right\}$

(56) Next Page

(56) $\sin 2x \tan^2 2x - \tan 2x = \sin 2x$

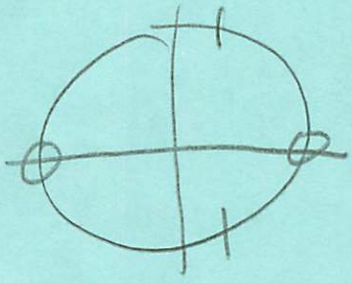
$$\sin 2x \left(\frac{\sin^2 2x}{\cos^2 2x} - \frac{1}{\cos 2x} - 1 \right) = 0$$

$$\frac{\sin 2x}{\cos^2 2x} \left(\frac{1 - \cos^2 2x - \cos 2x - \cos^2 2x}{} \right) = 0$$

$$-\frac{\sin 2x}{\cos^2 2x} \left(2\cos^2 2x + \cos 2x - 1 \right) = 0$$

$$-\frac{\sin 2x}{\cos^2 2x} \left(2\cos 2x - 1 \right)(\cos 2x + 1) = 0$$

$\cos 2x = \frac{1}{2} \quad \cos 2x = -1$



$2x \in \left\{ 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi, \frac{7\pi}{3}, 3\pi, \frac{11\pi}{3} \right\}$

$x \in \left\{ 0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6} \right\}$

(58)

$\tan^{-1}x + \tan^{-1}1 = \frac{3\pi}{4}$

$\tan^{-1}x + \frac{\pi}{4} = \frac{3\pi}{4}$

$\tan^{-1}x = \frac{\pi}{2}$

Reject.

Range of $\tan^{-1}x$ $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore \phi$

(56) previous page

$\sin 2x \cos^2 2x - \cos 2x = \sin 2x$

$\frac{\sin^3 2x}{\cos^2 2x} - \frac{\sin 2x}{\cos 2x} = \sin 2x$

$\sin 2x\left(\frac{\sin^2 2x}{\cos^2 2x} - \frac{1}{\cos 2x} - 1\right) = 0$

$\sin 2x\left(\frac{1-\cos^2 2x}{\cos^2 2x} - \frac{1}{\cos 2x} - 1\right) = 0$

$\sin 2x\left(\frac{1}{\cos^2 x} - 1 - \frac{1}{\cos 2x} - 1\right) = 0$

$-2\sin 2x = 0$

$2x \in \{0, \pi, 2\pi, 3\pi\}$

$x \in \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$.

(57) $\sin x + \cos x = 1$

exclude with 37/57

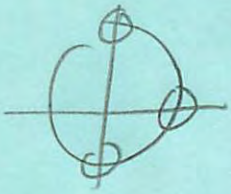
$\sin x = 1 - \cos x$

$\sin^2 x = 1 - 2\cos x + \cos^2 x$

$1 - \cos^2 x =$

$2\cos^2 x - 2\cos x = 0$

$2\cos x(\cos x - 1) = 0$



CK

0 $\sin 0 + \cos 0 = 1$

$0 + 1 = 1$ OK

$\frac{\pi}{2}$ $\sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1$

$1 + 0 = 1$ OK

$\frac{3\pi}{2}$ $\sin\frac{3\pi}{2} + \cos\frac{3\pi}{2} = 1$

$-1 + 0 = 1$ NO

$x \in \{0, \frac{\pi}{2}\}$.

(58) $\sin 2x \sin x - \cos 2x \cos x = 1$

$\cos 3x = -1$

$3x \in \{\pi, 3\pi, 5\pi\}$

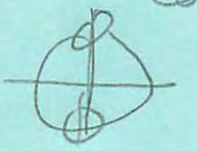
$x = \{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\}$.

(59) $2\sin(x - \frac{\pi}{6}) = \sqrt{3}\sin x$

$2(\sin x \cos\frac{\pi}{6} - \sin\frac{\pi}{6}\cos x) = \sqrt{3}\sin x$

$\sqrt{3}\sin x - \cos x = \sqrt{3}\sin x$

$\cos x = 0$



$x \in \{\frac{\pi}{2}, \frac{3\pi}{2}\}$.

(60) $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$x \in \{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

wrong ans $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{3\pi}{2}$

(61) $\sin^2(x + \frac{\pi}{2}) + \cos^2(x + \frac{\pi}{2}) - 1 = 0$

$x \in \mathbb{R}$ Horton identity.

$0 \le x < 2\pi$.

(62) $2\cos 2x + \sin 2x \sec x = 0$

$2\cos^2 x - 2\sin^2 x + 2\sin x = 0$

$2 - 4\sin^2 x + 2\sin x = 0$

$2\sin^2 x - \sin x - 1 = 0$

Now same as (60)

$x \in \{\cancel{\frac{\pi}{2}}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$.

But Has Sec x so Reject $\frac{\pi}{2}$,

wrong answer $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(63) $3\sec x - \cos x = 2$

$3 - \cos^2 x = 2\cos x$

$\cos^2 x + 2\cos x - 3 = 0$

$(\cos x - 1)(\cos x + 3) = 0$

reject

$x = 0.$

(64) $2\cos^2 x \sin^2 x - \sin x \cos x = 0$

$\sin x \cos x (2\sin x \cos x - 1) = 0$

$\sin x \cos x (\sin 2x - 1) = 0$

$2x \in \left\{\frac{\pi}{2}, \frac{5\pi}{2}\right\}$

$x \in \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

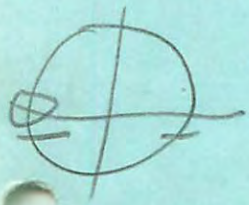
$x \in \left\{0, \frac{\pi}{4}, \frac{\pi}{2}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}\right\}.$

(65) $\sin 2x + \cos x + 2\sin x = -1$

$2\sin x \cos x + \cos x + 2\sin x + 1 = 0$

$\cos x (2\sin x + 1) + 2\sin x + 1 = 0$

$(2\sin x + 1)(\cos x + 1) = 0$



$x \in \left\{\pi, \frac{2\pi}{6}, \frac{11\pi}{6}\right\}$

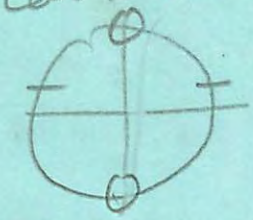
(66) $\sin(\pi - 2x) = \cos x$

$\sin\pi\cos 2x - \sin 2x \cos\pi = \cos x$

$0 + \sin 2x - \cos x = 0$

$2\sin x \cos x - \cos x = 0$

$\cos x (2\sin x - 1) = 0$

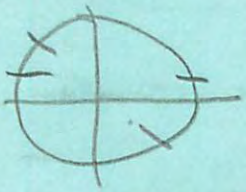


$x \in \left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

(67) $2\tan x \sin x + 2\sin x = \tan x + 1$

$2\sin x (\tan x + 1) - (\tan x + 1) = 0$

$(\tan x + 1)(2\sin x - 1) = 0$



$x \in \left\{\frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{4}\right\}$

(80) $\sin 2x - \sin 4x = 2\sin x$

$\sin 2x - 2\sin 2x \cos 2x = 2\sin x$

$2\sin x \cos x - 4\sin x \cos x (\cos^2 x - \sin^2 x) =$

$2\sin x \cos x - 4\sin x \cos x (2\cos^2 x - 1) - 2\sin x = 0$

$-2\sin x (-\cos x + 2\cos x (2\cos^2 x - 1) - 1) = 0$

$-2\sin x (4\cos^3 x - 3\cos x - 1) = 0$

$-2\sin x (\cos x + 1)(4\cos^2 x - 4\cos x + 1) = 0$

$-2\sin x (\cos x + 1)(2\cos x - 1)^2 = 0$

$\cos x + 1 \;\big|\; 4\cos^3 x - 3\cos x - 1$

$4\cos^2 x - 4\cos x + 1$

$4\cos^3 x +$

$4\cos^2 x$

$4\cos^2 x - 4\cos x$

$\cos x - 1$

$x \in \left\{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}.$

(68) $\tan\left(\pi + \arcsin\frac{2}{3}\right) = x$

$$\frac{\tan\pi + \tan\left(\sin^{-1}\frac{2}{3}\right)}{1 - \tan\pi\tan\sin^{-1}\frac{2}{3}} = x$$

$$\frac{0 + \frac{\sin\left(\sin^{-1}\frac{2}{3}\right)}{\cos\left(\sin^{-1}\frac{2}{3}\right)}}{1 - 0} = x$$

$$\frac{\frac{2}{3}}{\sqrt{1-\frac{4}{9}}} = x$$

$$\frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}} = x$$

$$x = \frac{2\sqrt{5}}{5}$$

(69) $\sin x = \cos(2x - \pi)$

$= \cos 2x \cos\pi + 0$

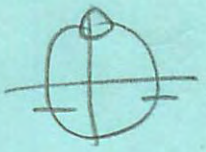
$= -\cos^2 x + \sin^2 x$

$= -1 + 2\sin^2 x$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$x \in \left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$



(70) $\sin 2x \sin x + \cos x \cos 2x = 1$

$\cos(2x - x) = 1$

$\cos x = 1$

$x = 0$

(71) $\cos 3x + \cos x = \cos 2x$ Replaced

$\cos 2x \cos x - \sin 2x \sin x + \cos x - \cos 2x = 0$

$(\cos^2 x - \sin^2 x)\cos x - 2\sin^2 x \cos x + \cos x - \cos^2 x + \sin^2 x = 0$

$\cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x + \cos x - \cos^2 x + \sin^2 x = 0$

$\cos^3 x - 3\sin^2 x \cos x - \cos^2 x + \sin^2 x = 0$

$\cos^3 x - 3\cos x(1 - \cos^2 x) - \cos^2 x + 1 - \cos^2 x = 0$

$\cos^3 x - 3\cos x + 3\cos^3 x - 2\cos^2 x + 1 = 0$

$4\cos^3 x - 2\cos^2 x - 3\cos x + 1 = 0$

$(\cos x - 1)(4\cos^2 x + 2\cos x - 1) = 0$

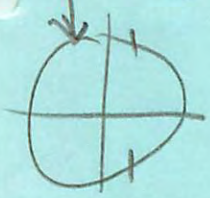
$\cos x = \frac{-2 \pm \sqrt{21}}{8}$

$$\begin{array}{r|l} & 4c^2 + 2c - 1 \\ \hline c - 1 & 4c^3 - 2c^2 - 3c + 1 \\ & 4c^3 - 4c^2 \\ \hline & 2c^2 - 3c \\ & 2c^2 - 2c \\ \hline & -c + 1 \\ & -c + 1 \\ \hline & \end{array}$$

(71) $\cos^2(x+\frac{\pi}{3}) - \sin^2(x+\frac{\pi}{3}) = \frac{1}{2}$

start

$\cos 2(x+\frac{\pi}{3}) = \frac{1}{2}$



$2(x+\frac{\pi}{3}) = \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}$

$x+\frac{2\pi}{6} = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}$

$0 \le x < 2\pi$

$\frac{\pi}{3} < x+\frac{\pi}{3} < 2\pi+\frac{\pi}{3}$

$x = \{\frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\}$.

(72) $2+\sec x = 0$

$\sec x = -2$

$\cos x = -\frac{1}{2}$

$x = \{\frac{2\pi}{3}, \frac{4\pi}{3}\}$

(73) $\tan(\tan^{-1}x + \tan^{-1}1) = -3$

$$\frac{\tan(\tan^{-1}x) + \tan(\tan^{-1}1)}{1 - \tan(\tan^{-1}x)\tan(\tan^{-1}1)} = -3$$

$$\frac{x+1}{1-x} = -3$$

$x+1 = -3+3x$

$4 = 2x$

$x = 2$

wrong ans -2

(74) $\tan(\cos^{-1}u) = x$ Next page

$\frac{\sin(\cos^{-1}u)}{\cos(\cos^{-1}u)} = x$

$\frac{\sqrt{1-u^2}}{u} = x$

(75) $\cos(\sin^{-1}u) = x$ Next page

$x = \sqrt{1-u^2}$

(76)

$\cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{2} - \sin^{-1}3x$

$\cos(\cos^{-1}\frac{\sqrt{3}}{2}) = \cos(\frac{\pi}{2} - \sin^{-1}3x)$

$\frac{\sqrt{3}}{2} = \cos\frac{\pi}{2}\cos(\sin^{-1}3x) + \sin\frac{\pi}{2}\sin(\sin^{-1}3x)$

$0 \qquad\qquad 1$

$\frac{\sqrt{3}}{2} = 3x$

$x = \frac{\sqrt{3}}{6}$

(77) $\cos(\frac{1}{2}\arctan\frac{3}{4}) = x$

$\cos\frac{1}{2}x = \pm\sqrt{\frac{1}{2}(1+\cos x)}$

$\cos(\tan^{-1}\frac{3}{4}) = \frac{1}{\sqrt{1+\frac{9}{16}}}$

$= \frac{1}{\sqrt{\frac{25}{16}}}$

$= \frac{4}{5}$

$\pm\sqrt{\frac{1}{2}(1+\frac{4}{5})}$

$\pm\sqrt{\frac{9}{10}}$

$\pm\frac{3\sqrt{10}}{10} = x$

(78) $\sin(\cos^{-1}u) = x$

$x = \sqrt{1-u^2}$

(79) $\cos\left(\arcsin\frac{1}{2} + \cos^{-1}\frac{5}{13}\right) = x$

$\cos(\sin^{-1}\frac{1}{2})\cos\cos^{-1}\frac{5}{13} - \sin(\sin^{-1}\frac{1}{2})\sin(\cos^{-1}\frac{5}{13}) = x$

$\sqrt{1-\frac{1}{4}} \cdot \frac{5}{13} - \left(\frac{1}{2}\right)\sqrt{1-\frac{25}{169}} = x$

$\frac{\sqrt{3}}{2} \cdot \frac{5}{13} + \frac{1}{2} \cdot \frac{12}{13} = x$

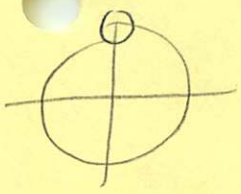
$\frac{5\sqrt{3}+12}{26}$

(80) Sadlers Problem 67

1995

(74) $\sin 2x\cos 3x + \cos 2x \sin 3x = 1$

$\sin 5x = 1$



$5x = \{\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \frac{17\pi}{2}\}$

$x = \{\frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10}\}$

(75) $\tan^2 3x = 1$

$\tan 3x = \pm 1$

$3x = \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}, \frac{17\pi}{4}, \frac{19\pi}{4}, \frac{21\pi}{4}, \frac{23\pi}{4}$

$x = \{\frac{\pi}{12}$ etc

(78) $2\cos 2x = \sqrt{3}$

$\cos 2x = \frac{\sqrt{3}}{2}$

$2x = \{\frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}\}$

$x = \{\frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}\}$

$0, \frac{2\pi}{3}, \frac{4\pi}{3}$

(80) $\sin 2x \tan^2 2x - \tan 2x = \sin 2x$ old problem 56

(86) $\cos x = \cos 2x$

$\cos x = \cos^2 x - \sin^2 x$

$\cos x = 2\cos^2 x - 1$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

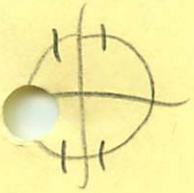
$\cos x = -\frac{1}{2}$ $\cos x = 1$

$3x = \{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3},$

$\frac{13\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \frac{17\pi}{3}$

(87) $\tan^2 3x = 3$

$\tan 3x = \pm\sqrt{3}$



$x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$ etc

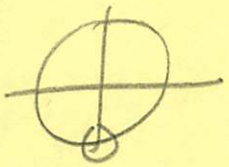
(90) $\sqrt{3}\tan x = 2\sin x \tan x$

$\tan x(\sqrt{3} - 2\sin x) = 0$

$\tan x = 0 \quad \sin x = \frac{\sqrt{3}}{2}$

$x \in \{0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi\}$

(93) $\sin^3 5x = -1$



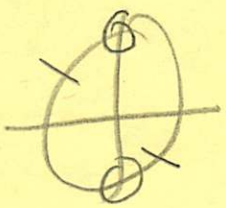
$5x = \{\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \frac{15\pi}{2}, \frac{19\pi}{2}\}$

$x \in \{\frac{3\pi}{10}, \frac{7\pi}{10}, \frac{11\pi}{10}, \frac{3\pi}{2}, \frac{19\pi}{10}\}$

(47) $2\cot^2 x + 2\cot x = 0$

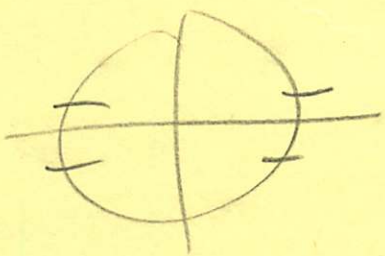
$2\cot x(\cot x + 1) = 0$

$\cot x = 0 \quad \cot x = -1$



$x \in \{\frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\}$

(49) $\cot x \tan 2x = 3$



$\cot x \cdot \frac{2\tan x}{1 - \tan^2 x} = 3$

$2 = 3 - 3\tan^2 x$

$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm \frac{1}{\sqrt{3}}$

$x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

FREEDOM COVER SHEET

Name _____

Per _____

Score

1) 19	2) 18	3) 17	4) 16
5) 15	6) 14	7) 13	8) 12
9) 11	10) 10	11) 9	12) 8
13) 7	14) 6	15) 5	16) 4
17) 3	18) 2	19) 1	20) 0
21) −1	22) −2	23) −3	24) −4
25) −5	26) −6	27) −7	28) −8
29) −9	30) −10	31) −11	32) −12
33) −13	34) −14	35) −15	36) −16
37) −17	38) −18	39) −19	40) −20
41)	42)	43)	44)
45)	46)	47)	48)
49)	50)	51)	52)
53)	54)	55)	56)
57)	58)	59)	60)
61)	62)	63)	64)
65)	66)	67)	68)
69)	70)	71)	72)
73)	74)	75)	76)
77)	78)	79)	80)
81)	82)	83)	84)
85)	86)	87)	88)
89)	90)	91)	92)
93)	2 bonus = 1 —		

FREEDOM COVER SHEET

Name _____

Per _____

Score

1) 19	2) 18	3) 17	4) 16
5) 15	6) 14	7) 13	8) 12
9) 11	10) 10	11) 9	12) 8
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17) 3	18) 2	19) 1	20) 0
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25) −5	26) −6	27) −7	28) −8
29) −9	30) −10	31) −11	32) −12
33) −13	34) −14	35) −15	36) −16
37) −17	38) −18	39) −19	40) −20
41)	42)	43)	44)
45)	46)	47)	48)
49)	50)	51)	52)
53)	54)	55)	56)
57)	58)	59)	60)
61)	62)	63)	64)
65)	66)	67)	68)
69)	70)	71)	72)
73)	74)	75)	76)
77)	78)	79)	80)
81)	82)	83)	84)
85)	86)	87)	88)
89)	90)	91)	92)
93)			

2 bonus = 1 —

Write # wrong and put in upper left corner of student's coversheet.

"FREEDOM" COVER SHEET

Name _____

Per _____ Date _____

1) _____ 2) _____ 3) _____ 4) _____

5) _____ 6) _____ 7) _____ 8) _____

9) _____ 10) _____ 11) _____ 12) _____

13) _____ 14) _____ 15) _____ 16) _____

17) _____ 18) _____ 19) _____ 20) _____

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25) _____ 26) _____ 27) _____ 28) _____

29) _____ 30) _____ 31) _____ 32) _____

33) _____ 34) _____ 35) _____ 36) _____

37) _____ 38) _____ 39) _____ 40) _____

41) _____ 42) _____ 43) _____ 44) _____

45) _____ 46) _____ 47) _____ 48) _____

49) _____ 50) _____ 51) _____ 52) _____

53) _____ 54) _____ 55) _____ 56) _____

57) _____ 58) _____ 59) _____ 60) _____

61) _____ 62) _____ 63) _____ 64) _____

65) _____ 66) _____ 67) _____ 68) _____

69) _____ 70) _____ 71) _____ 72) _____

73) _____ 74) _____ 75) _____ 76) _____

77) _____ 78) _____ 79) _____ 80) _____

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85) _____ 86) _____ 87) _____ 88) _____

89) _____ 90) _____ 91) _____ 92) _____

93) _____